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Simultaneous Estimation of Aquifer Parameters and Original Hydrocarbons in Place From Production Data Using Numerical Inversion of Laplace Transform

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Abstract

A new method is presented for the simultaneous estimation of original hydrocarbons in place OHIP and aquifer parameters for a circular aquifer surrounding a hydrocarbon reservoir. Water influx data are represented by the van-Everdingen and Hurst (VEH) unsteady state solution. In addition to the initial oil and/or gas in place, the aquifer parameters are also estimated. These parameters include the water influx constant B , the relative aquifer size R_{eD} , and the time adjustment factor c which transforms real time t into dimensionless time t_D . The parameters B and c can be used to estimate the storativity $h\phi C_i$ and the transmissibility kh/μ of the aquifer.

The method of least squares is applied to the material balance equation to estimate the parameters resulting in a non-linear regression problem. Because the solution in Laplace space is simpler than the solution in the real time domain, numerical inversion of Laplace transform was used to obtain the partial derivatives of the VEH solution with respect to aquifer parameters needed for least squares method. Nonlinear regression analysis using numerical inversion of Laplace transform is applied. The Levenberg method was used for parameter estimation to guarantee convergence when the initial guess is not close enough to the correct solution.

The model is linear with respect to original hydrocarbons in place N , G_i , and the water influx constant B , but is nonlinear with respect to the dimensionless aquifer size R_{eD} and time adjustment factor c which transforms real time t into dimensionless time t_D . Assuming the values for c and R_{eD} enables the calculation of N , G_i , and B and hence the estimation of the sum of squares of the residuals. Maps of the sum of squares of the residuals are generated and displayed to show regions of maxima and minima in the aquifer parameters space. Maps explain the non-uniqueness of the solution usually encountered in nonlinear least squares. The knowledge

of the approximate values of some of the aquifer parameters from geological and engineering data will help choosing the proper initial guess to achieve convergence to the correct solution

Introduction

The estimation of initial hydrocarbons (oil and/or gas) in place OHIP and the prediction of future reservoir performance are of great importance for the development of these reservoirs. Volumetric methods based on geological and seismic data can be used to estimate the OHIP. With a reasonable estimates of OHIP, the material balance equation MBE can be used to predict future reservoir performance for volumetric reservoirs (no water influx). For non-volumetric (water drive) reservoirs, the aquifer parameters are needed. If enough production data are available for a given reservoir, the MBE can be used to estimate the OHIP and aquifer parameters.

For volumetric reservoirs (no water influx), the MBE is linear the parameters N and G . In this case, the MBE represent an equation of a plane. Havlena and Odeh¹ showed how it can be arranged as an equation of a straight line by grouping production and pressure dependent terms. Tehrani², however, indicated that regression should be performed on the original (non-grouped) MBE to preserve the physical meaning of regression variables. In these cases, linear or multiple regression analysis by the method of least squares is used to estimate the original oil in place N and the gas cap ratio m for oil reservoirs or the original gas in place G_i for gas reservoirs.

For non-volumetric (water drive) reservoirs the material balance equation can be used to estimate both OHIP and aquifer parameters. An aquifer model describing water influx from the aquifer into the reservoir is needed. In most field cases, such model is nonlinear. The van-Everdingen and Hurst's (VEH) unsteady state model³ is an exact analytical solution for circular aquifers with homogeneous properties. The model is linear with respect to the water influx constant B , but nonlinear with respect the dimensionless aquifer size R_{eD} and time adjustment factor c which transforms real time t into dimensionless time t_D . It is therefore obvious that linear regression can not be used directly to estimate both OHIP and aquifer parameters B , c , and R_{eD} .

To overcome the nonlinearity problem, most investigators used some kind of a trial and error approach. In such cases, values for aquifer parameters are assumed and linear regression is performed to estimate N and m for oil or G_i for gas reservoirs. The standard deviation or the sum of squares of

errors is calculated. The values of aquifer parameters that yield the smallest sum of squares of errors or fit a straight line in the Havlena-Odeh method are selected. Most investigators used the tabulated values of $Q(t_D)$ given by Van-Everdingen and Hurst to calculate the water influx.

Dougherty *et al.*⁴ used tabulated values of the solution obtained by Wattenbarger and Ramey⁵ to get the error terms. By performing different runs within the limits of parameters, they expressed the sum of squares of errors as a second order polynomial in the parameters and performed the minimization to obtain the optimum values of the parameters.

Chen, Chen and Lin⁶ used polynomial approximations of $Q(t_D)$ by Klins *et al.*⁷ to evaluate the water influx and used the simplex search method for parameter estimation.

Chatas and Malekfan⁸ proposed the application of nonlinear regression to the Van-Everdingen and Hurst solution in real space to estimate aquifer parameters. They outlined the procedure and evaluated the partial derivatives but did not present actual solutions.

The VEH solution in real time space is very complex and computational and convergence problems may be encountered. The solution in Laplace space is simpler and the use of the numerical inversion of the Laplace transform makes it easier to evaluate the first and second derivatives with respect to the aquifer parameters. This approach is applied in this paper.

This method provides simultaneous estimation of original hydrocarbons in place OHIP and aquifer parameters (the water influx constant B , the relative aquifer size R_{eD} , and the time adjustment factor c relative aquifer size R_{eD}) from field production data. The storativity $h\phi C_i$ and the transmissibility kh/μ of the aquifer can then be estimated. These parameters are necessary for performance prediction of oil and/or gas reservoirs performance by material balance equation.

Theoretical Considerations

The conventional form of the MBE can be following in the following form

$$N_p(\beta_o - R_s\beta_g) + G_p\beta_g + W_p\beta_w = N[\beta_o - \beta_{oi} + (R_{si} - R_s)\beta_g] + G_i(\beta_g - \beta_{gi}) + We \tag{1}$$

Equation (1) can be used for undersaturated oil reservoirs with $G_i = 0$ or for gas reservoirs with $N=0$, $N_p=0$.

In simple models, the water influx from the aquifer into the reservoir is treated using Schilthuis steady state model⁹.

$$We = K \int_0^t (p_i - p) dt \tag{2}$$

This makes the material balance equation linear in the parameters N , G_i and K taking the following form.

$$y = N X_1 + G_i X_2 + K X_3 \tag{3}$$

Equation (3) represents a hyper plane and multiple regression analysis can be used to estimate the three parameters N , G_i and K from production and PVT data.

The steady state water influx model however rarely describes the actual behavior of aquifers. An exact

mathematical solution of the diffusivity equation is given by van Everdingen and Hurst³ for a radial flow system at constant terminal pressure. The solution is given by

$$We = B\Delta p Q(t_D) \tag{4}$$

where for We in Bbl, h and r_w in ft, K in md and t in days

$$B = 1.119h\phi C_i r_w^2 \tag{5}$$

$$t_D = \frac{.00634Kt}{\mu C_i \phi r_w^2} = ct \tag{6}$$

The dimensionless function $Q(t_D)$ is given by

$$Q(t_D) = \frac{R_{eD}^2 - 1}{2} - 2 \sum_{n=1}^{\infty} \frac{J_1^2(\alpha_n R_{eD}) e^{-\alpha_n^2 t_D}}{\alpha_n^2 [J_0^2(\alpha_n) - J_1^2(\alpha_n R_{eD})]} \tag{7}$$

where α_n are roots of the equation

$$J_1(\alpha_n R_{eD}) Y_0(\alpha_n) - Y_1(\alpha_n R_{eD}) J_0(\alpha_n) = 0 \tag{8}$$

For variable pressure case, the principle of superposition (convolution) is applied to estimate the cumulative water influx into the reservoir. The solution is given by

$$We(t_D) = B \int_0^{t_D} Q(t_D - \tau) \Delta P'(\tau) d\tau \tag{9}$$

Calculations of We Using SP and LP methods

For variable reservoir pressure, the pressure history is approximated into a number of constant pressure steps with discontinuous jumps at the data points as shown in fig. 2. The integration in Eq. (9) is then approximated by a summation as follows

$$We(k) = B \sum_{j=1}^k \Delta P_j Q[t_D(k) - t_D(j-1)] \tag{10}$$

This method is called the step pressure (SP) method.

Vogt and Wang¹⁰ suggested approximating the pressure behavior by a series of linear segments connecting successive data points. This is expected to give a more accurate representation of the pressure history. The basis for this method is to replace $\Delta P'$ in Eq. (9) by the slope m and integrating by parts to obtain

$$We(t_D) = B \int_0^{t_D} \tilde{Q}(t_D - \tau) \frac{dm}{d\tau}(\tau) d\tau \tag{11}$$

where

$$\tilde{Q}(t_D) = \int_0^{t_D} Q(\tau) d\tau \tag{12}$$

Equation (11) is then approximated by a summation to

$$We(k) = B \sum_{j=1}^k \Delta m_j \tilde{Q}[t_D(k) - t_D(j-1)] \tag{13}$$

This method is called the linear pressure (LP) method. It is to be noticed that the slope m in Eq.(11) and Eq.(13) is calculated w.r.t the dimensionless time t_D .

Results indicated that the LP method is more accurate than the SP method and will be used exclusively for parameter estimation in this paper.

Numerical Inversion of Laplace Transform

The complexity of using equation (7) in computing the water influx We is apparent. First Eq. (8) must be solved iteratively for enough numbers of successive roots α_n . The summation in Eq.(7) is to be continued until convergence of the infinite series is achieved. The problem is further complicated in the parameter estimation problem where in addition to evaluating We , the derivatives of $Q(t_D)$ must also be evaluated. These difficulties prompted the investigation of the possibility of performing evaluation and optimization in Laplace space using the Stehfest algorithm¹¹ for the numerical inversion of Laplace transform.

In Laplace space, the expression for the transform of the dimensionless water influx $\bar{Q}(s)$ is

$$\bar{Q}(s) = \frac{I_1(\sqrt{s}R_{eD})K_1(\sqrt{s}) - K_1(\sqrt{s}R_{eD})I_1(\sqrt{s})}{s^{3/2}[K_1(\sqrt{s}R_{eD})I_0(\sqrt{s}) + I_1(\sqrt{s}R_{eD})K_0(\sqrt{s})]} \dots\dots\dots (14)$$

The inverse of $\bar{Q}(s)$ by the Stehfest algorithm is

$$Q(t_D) = l^{-1}[\bar{Q}(s)] = \frac{\ln 2}{t_D} \sum_{i=1}^N V_i \bar{Q}(s = \frac{i \ln 2}{t_D}) \dots (15)$$

In Laplace space, using rules for transform of the integrals,

$$\bar{Q}(s) = l[\tilde{Q}(t_D)] = \frac{\tilde{Q}(s)}{s} \dots\dots\dots (16)$$

where $\bar{Q}(s)$ is given by Eq. (14)

Aquifer Parameters

From Eq. (4)-(7) , the parameters needed to evaluate the water influx We at a given time t are the aquifer constant B , the relative aquifer radius $R_{eD}=r_e/r_w$ and the constant c in Eq. (6) which transforms real time t into dimensionless time t_D .

The parameters B and c can be expressed in terms of the commonly used parameters of transmissibility T (Kh/μ) and storativity S ($h\phi c_t$) as follows

$$B = 1.119S r_w^2 \dots\dots\dots (17)$$

$$c = .00634 \frac{T}{S r_w^2} \dots\dots\dots (18)$$

Moreover, the parameter B can be related to the reservoir pore volume V_p as follow

$$B = 2C_t V_p \dots\dots\dots (19)$$

Sills¹² related the parameter B to the original oil in place N

$$B = 2C_t V_p = 2C_t \frac{N\beta_{oi}}{(1 - S_{wi})} \dots\dots\dots (20)$$

Similarly for an oil reservoirs with a gas cap,

$$B = 2C_t \frac{(1+m)N\beta_{oi}}{(1 - S_{wi})} \dots\dots\dots (21)$$

These relations reduces the number of parameters to be estimated by one parameter by making B and N , G , or $N(1+m)$ dependent. Although this may sound attractive mathematically, its physical validity is questionable. The term V_p in Eq. (14), (15) represent all the pore volume present in the reservoir within the radius r_w and thickness h including any shales and nonconnected pore volume. The term N in the material balance equation represents the oil subjected to expansion due to pressure change which includes only oil present in interconnected pore space. It is therefore important to have B and N or G_i as two independent variables when applying regression analysis for parameter estimation to the material balance equation.

Aquifer Parameter Estimation

The MBE, Eq.(1), using Eqs. 13, 15, and 16 , can be written in the following form

$$Y_k = NX_{1k} - G_i X_{2k} - B \sum_{j=1}^k \Delta m_j \frac{\ln 2}{c^2 [t_k - t_{j-1}]} \sum_{i=1}^N V_i \bar{Q}(\frac{i \ln 2}{c[t_k - t_{j-1}]}) \dots\dots\dots (22)$$

where

$$Y = N_p (\beta_o - R_s \beta_g) + G_p \beta_g + W_p \beta_w \dots\dots\dots(23)$$

$$X_1 = \beta_o - \beta_{oi} + (R_{si} - R_s) \beta_g \dots\dots\dots(24)$$

$$X_2 = \beta_g - \beta_{gi} \dots\dots\dots(25)$$

In Eq.(26), the slope m is evaluated w.r.t the real time t which is c times the slope w.r.t the dimensionless time t_D .

Given production and average pressure drop in the reservoir ΔP at different times, the method of least squares is applied to estimate OHIP and aquifer parameters. The method is used to find the values of N , G_i . B , C and R_{eD} which minimize the sum of squares (SSE) of the errors (residuals) of the MBE using the five parameters for all data points.

Investigation of Eq. (22) shows that the term Y_k represents total fluid withdrawal (production) from the reservoir at time interval k while the right side of the equation represents the contributions of the different drive mechanisms to the production. This side can be considered as the value of fluid withdrawal from the reservoir calculated by using the aquifer parameters and the original hydrocarbons in place. Since both terms and so the difference between them runs into millions of barrels, the SSE is expected to be very large. Also the values of these terms increase with time as the cumulative production increases and so their relative contribution to the SSE will be different for different points.

To overcome this problem, the method of weighted least squares is used. The residuals are normalized by dividing each term by the fluid withdrawal at that point, Y_k .

A relative difference of 1% in the residuals will result in a sum of squares of 1.0E-4 multiplied by the number of data points. A convergence criterion of $SSE = 1.0E-5$ amounts to a relative error of 0.1% for a run of 10 data points.

So it is required to minimize the following objective function

$$SSE(B, c, R_{eD}) = \sum_{k=1}^{Nt} R_{s_k}^2 \dots\dots\dots (26)$$

where the relative residual (error) R_s is given by

$$R_{s_k} = \left[\frac{Y_k - NX_{1k} - G_i X_{2k}}{-B \sum_{j=1}^k \Delta m_j \frac{\ln 2}{c^2 [t_k - t_{j-1}]} \sum_{i=1}^N V_i \bar{Q} \left(\frac{i \ln 2}{c [t_k - t_{j-1}]} \right)} \right] / Y_k \dots\dots\dots (27)$$

The minimality conditions for the objective function are:

$$g_i = \frac{1}{2} \frac{\partial SSE}{\partial \theta_i} = \sum_{k=1}^{Nt} [R_s \frac{\partial R_s}{\partial \theta_i}]_k = 0 \dots\dots\dots (28)$$

Where θ is the vector of parameters to be estimated.

$$\theta = [N, G_i, B, c, Re_D]^T \dots\dots\dots (29)$$

The set of Equations (31) are the normal equations of the system and must be solved simultaneously for parameters θ .

From Eq. (30), the derivatives of the residual R_{s_k} w.r.t. the parameters N, G_i, B, c and R_{eD} are given by

$$\frac{\partial R_{s_k}}{\partial N} = -X_{1k} / Y_k \dots\dots\dots (30)$$

$$\frac{\partial R_{s_k}}{\partial G_i} = -X_{2k} / Y_k \dots\dots\dots (31)$$

$$\frac{\partial R_{s_k}}{\partial B} = -\sum_{j=1}^k \Delta m_j \frac{\ln 2}{c^2 [t_k - t_{j-1}]} \sum_{i=1}^N V_i \bar{Q} \left(\frac{i \ln 2}{c [t_k - t_{j-1}]} \right) / Y_k \dots\dots\dots (32)$$

$$\frac{\partial R_{s_k}}{\partial R_{eD}} = -B \sum_{j=1}^k \Delta m_j \frac{\ln 2}{c^2 [t_k - t_{j-1}]} \sum_{i=1}^N V_i \frac{\partial}{\partial R_{eD}} \left[\bar{Q} \left(\frac{i \ln 2}{c [t_k - t_{j-1}]} \right) \right] / Y_k \dots\dots\dots (33)$$

$$\frac{\partial R_{s_k}}{\partial c} = -\frac{B}{Y_k} \sum_{j=1}^k \Delta m_j \frac{\ln 2}{[t_k - t_{j-1}]} \left[\sum_{i=1}^N \frac{V_i}{c^2} \left\{ \frac{\partial}{\partial c} \left[\bar{Q} \left(\frac{i \ln 2}{c [t_k - t_{j-1}]} \right) \right] \right\} - \frac{2}{c} \bar{Q} \left(\frac{i \ln 2}{c [t_k - t_{j-1}]} \right) \right] \dots\dots\dots (34)$$

Since the equations are nonlinear, an iterative procedure must be used. The application of the multi-variable Newton Rapson method to the normal equations results in the well known Gauss method of optimization. The solution at any iterative step is given in a matrix form as

$$H \delta \theta = -g \dots\dots\dots (35)$$

Where H_{ij} 's are the elements of the Hessian matrix which are the partial derivatives of the normal functions g_i with respect to parameter θ_j and the vector $\delta \theta$ is the changes in the values of the parameters in the iteration step. From Eq. (26)

$$H_{ij} = \frac{1}{2} \frac{\partial g_i}{\partial \theta_j} = \sum_{k=1}^{Nt} \left[\frac{\partial R_s}{\partial \theta_i} \frac{\partial R_s}{\partial \theta_j} + R_s \frac{\partial^2 R_s}{\partial \theta_i \partial \theta_j} \right]_k \dots\dots\dots (36)$$

differentiation of Eq. (29) - (33), we get

$$\frac{\partial}{\partial \theta_j} \frac{\partial R_s}{\partial N} = \frac{\partial}{\partial \theta_j} \frac{\partial R_s}{\partial G_i} = 0 \quad (j = 1 - 5) \quad \dots\dots\dots (37)$$

$$\frac{\partial}{\partial \theta_j} \frac{\partial R_s}{\partial B} = 0 \quad (j = 1 - 3) \quad \dots\dots\dots (38)$$

From Eq. (14) and (16)

$$\bar{Q}(s) = \frac{I_1(\sqrt{s} R_{eD}) K_1(\sqrt{s}) - K_1(\sqrt{s} R_{eD}) I_1(\sqrt{s})}{s^{5/2} [K_1(\sqrt{s} R_{eD}) I_0(\sqrt{s}) + I_1(\sqrt{s} R_{eD}) K_0(\sqrt{s})]} \dots\dots\dots (39)$$

Expressions for the derivatives of $\bar{Q}(s)$ with respect to parameters c and Re_D can be obtained from Eq.(39) by setting the Laplace transform parameter S to the value $[i \ln 2 / c (t_k - t_{j-1})]$. The derivation is straightforward and is given in the Appendix of reference 13. .

The Gauss method uses the two terms in Eq.(33) for the Hessian matrix and thus requires evaluation of second derivatives. A modified form which does not require evaluation of the second derivatives is the Gauss-Newton method. In this method, only the first term is used which involves products of first derivatives only. The justification for dropping the second term in the Hessian matrix is that the second derivatives in that term are multiplied by the residual R_s which is supposed to be very small close to the solution. This however might not be the case at points far from the solution. In this work we will consider the full form of the Hessian matrix since second derivatives can be evaluated analytically when the Stehfest algorithm is used for the numerical inversion of Laplace transform as shown in the appendix.

Although it is known that the Gauss method and the modified Gauss-Newton method have quadratic convergence near the solution, it may diverge if the initial guess is far from the solution. On the other hand, the method of steepest descent guarantees decreasing the objective function in each iteration but its convergence is slow. Methods presented by Levenberg and Marquardt are combinations of Gauss and the steepest descent methods. To guarantee that the objective function is decreasing, the Hessian matrix H must be positive definite. It can be made so by adding large positive numbers to its diagonal elements. Thus, the matrix H in Eq. (32) is replaced by $H + \lambda V$ where λ is a positive number and V is a diagonal matrix. Levenberg¹⁴ took V as the identity matrix I while a better choice suggested by Marquardt¹⁵ is to take the diagonal elements of V equal to the absolute values of the diagonal elements of H . So, the system of equations to be solved at each iteration becomes

$$[H + \lambda V] \delta \theta = -g \dots\dots\dots (40)$$

A very large value of $\lambda (\lambda \rightarrow \infty)$ is equivalent to the steepest descent method while $\lambda = 0$ represents to the Gauss method.

The system of the three linear equations represented by Eq. (34) can be solved by the Gauss elimination method or by Cramer's rule. The iteration is continued until a convergence criterion is achieved. Either the sum of squares of errors SSE or $\|g\|$, the norm of the vector g , is used.

$$\|g\| = \sum_{i=1}^5 g_i^2 \dots\dots\dots (41)$$

Solution Method

Investigation of Eqs.30 – 34 shows that the equations are linear in the parameters N , m , and B and only nonlinear in C and R_{eD} . Therefore, by assuming values of c and R_{eD} , Eqs. 30 - 32 can be solved for N , m , and B . Equations 33 and 34 can then be checked and corrected for c and R_{eD} . The equations will be satisfied only for the correct values of c and R_{eD} . For other values of c and R_{eD} , the Equations are not satisfied and the SEE will not be minimum. An error map can be generated by assuming values of c and R_{eD} , solving for N , m , and B and calculating the SSE. A region around the minimum value of SSE can be identified and used to choose values for the initial guess of c and R_{eD} so that the solution converges to the correct answer. It is to be noted that this procedure does not involve calculating second order derivatives of the objective function. This procedure is equivalent to a search method. However we will use this method to choose the initial guess for the iterative procedure of the nonlinear least squares method. Since the parameters have a wide range of values and are usually not known in advance, the proper choice of the initial guess of the iteration parameters is essential for the solution to converge to the correct value

Parameter estimation by the method of least squares (usually termed history matching) is known to exhibit non-uniqueness¹⁶. The procedure may converge to different solutions depending on the starting point of iteration (initial guess). It is therefore imperative to have an idea about the approximate values of the required parameters. This information can be obtained from geological and engineering data. The knowledge of such information combined with the calculated error maps can identify the ranges of initial guess that will achieve convergence of the parameter estimation routine.

A computer program is written using the developed procedure. The minimization was performed on the normalized sum of squares of residuals. This procedure will be illustrated by the given examples.

Results and Discussion

Example 1:

The assumed values for the aquifer parameters R_{eD} , B , and c and OHIP N , m are given in table 1. The reservoir performance was estimated using a modified version of Turner's method to account for the water influx. Water influx was calculated by the developed method using Eqs. 13 – 16. The PVT data was estimated using the following correlations:

$$R_g = 0.18737P + 104.63 \quad \dots\dots\dots (42)$$

$$B_o = 8 \times 10^{-5} P + 1.168 \quad \dots\dots\dots (43)$$

$$\frac{1}{B_g} = 0.062P - 4 \quad \dots\dots\dots (44)$$

The generated error map is shown in Fig. 1. Figures 2-4 show maps for the original oil in place N , the gas cap ratio m and the aquifer constant B respectively. It is seen from these maps that the contours for the assumed values of N , m , and B pass through the point $R_{eD}=6$ and $c = 0.4$ which are the assumed parameters. This point also lies on the SSE contour of 2.0×10^{-5} which the minimum contour is shown on the map although it is not the minimum value on the map. Figure 5 is an

overlay of the contours N , m , B with the assumed values. It is seen that the three contours intersect at the point $R_{eD}=6$ and $c = 0.4$. It can also be seen that two of the contours may coincide or intersect but the three contours do not intersect at any other point.

Inspection of fig. 1 shows that two regions, one with minimum and another with maximum values can be identified. It is clear that initial guesses to the left of the region of maximum values will not converge while guesses to the right of the region of minimum values and between the two regions will eventually converge to a solution in the region of minimum values.

Different starting points were chosen and the nonlinear regression was carried out. The results of these runs are shown in Table 2. The location of the points is shown on the SSE map in figure 1. Point 1 represents the assumed values of the aquifer. Points 2 and 7 are located to the right of the region of minimum values and are expected to converge. They however converge not to point 1 but to points on an axis in the centre of the minimum contour of 2×10^{-5} . Points 3 and 4 are located between the two regions of minima and are also expected to converge to the correct answer. Point 3 converges to a value very close to that of point 1 while point 4 converges to a point below point 1. Points 5 and 6 are in or to the left of the region of maximum values and did not converge as expected.

Example 2:

The second example used is taken from Dake¹⁷. The production history and PVT data for an oil reservoir with no gas cap is given in Table 9.3 of reference 16. Dake estimated the parameters B and c of the aquifer to be 6446 Bbl/psi and 5.671 year^{-1} respectively. The dimensionless aquifer radius R_{eD} was suspected to be 10. Dake calculated the water influx for the data points using Eq.(10) and the tabulated values of van Everdingen and Hurst for values of R_{eD} of 10 and 5 at the values of B and c given above. He then used the method of Havlena and Odeh to plot the data as a straight line. He found the data to plot as a straight line for $R_{eD}=5$ but not for $R_{eD}=10$. He thus concluded that the correct value of R_{eD} is 5 and not 10.

We used the production and PVT data to generate SSE map which is displayed in figure 6. Maps for N , M , and B are shown in figs. 7 through 9 respectively.

Inspection of figure 6 shows that four regions of local minima exist at values of R_{eD} of about 3,4,5, and 6 in the mapped space at different values of the parameter c . The values of R_{eD} and c chosen by Dake (point 1) lies apparently in one of these regions. If another value of c was chosen, a different value of R_{eD} other than 5 would better fit the data. Different points were chosen as shown in figure 6 and the results of regression analyses are shown in Table 3. Points 5 and 6 are located within two regions of local minima and they converge to values very close to their original values giving values of N of 307 and 321 MM STB as compared to 312 MM STB reported by Dake from volumetric method. The values for the gas cap ratio m was a small -ive number (-0.02) indicating no gas cap. The values of B are different from that reported by Dake. This is expected since the parameter B can be related to the parameter c (Eqs. 17, 18) which was different from that used by Dake. Point 3 converged to a point close to point 1 as can be expected from the gradient of the SEE map

in the neighborhood of the point. Points 2 and 4 do not seem to converge. The final results for these two points show smaller value for N (240-245 MM STB), a relatively large gas cap ratio (.37) and a negative aquifer parameter B .

It is therefore seen that the solution of the nonlinear regression analysis for parameter estimation based on least squares method is not unique. Different answers may be obtained depending on the initial guess of the parameters. In the method developed here we reduced the number of parameters to be guessed to two instead of five, namely C . The values of the other three parameters N , m , B can be determined from three linear algebraic equations.

So if a fairly reasonable estimate of the aquifer parameters R_{eD} and c can be made, the developed procedure would converge to the correct values. The time adjustment factor c depends on the permeability K , the viscosity μ , the porosity ϕ , the total compressibility C_t , and the aquifer radius r_w . All these properties can be estimated with reasonable accuracy (within $\pm 50\%$) of the actual values. The same can also be stated for the dimensionless aquifer radius R_{eD} . If the initial guess was made within these limits, the procedure is then expected to converge to the neighborhood of the true value.

Conclusions

The following conclusions can be made

1. A method is presented for simultaneous estimations of aquifer parameters and OHIP by the method of least squares applied to unsteady state van Everdingen and Hurst solution in Laplace domain. The aquifer parameters estimated are the relative aquifer size R_{eD} , the aquifer constant B and the time adjustment factor c in addition to the original oil in place N and/or the original gas in place G_i (gas cap ratio m). The pressure history is approximated by a series of linear segments (LP method) rather than the stair-like pressure steps (SP method). The LP method was found to give more accurate results.
2. Stehfest algorithm for numerical inversion of Laplace transform was used to evaluate the water influx and the first and second derivatives of the objective function with respect to aquifer parameters B , C , and R_{eD} . The Levenberg-Marquardt method was used in parameter estimation to avoid divergence of the solution if the initial guess is not close enough to the correct answer.
3. Maps for the sum of squares of errors SSE were constructed and used to identify the regions of initial guess that achieve convergence to the correct solution. Also maps for the parameters N , m , B are obtained. An overlay of these maps will show the approximate solution.
4. The use of additional geological and engineering information combined with the constructed maps will help in choosing the proper initial guess to achieved convergence to a solution in the neighborhood of the real parameters.

Nomenclature

B	= aquifer constant, bbl/psi [m^3/kPa]
c	= time adjustment factor, $\text{day}^{-1}[\text{s}^{-1}]$
C_t	= total formation compressibility, $\text{psi}^{-1} [\text{kPa}^{-1}]$
G_i	= initial gas in place, SCF [st m^3]
G_p	= cumulative gas production, SCF [st m^3]
H	= Hessian matrix
I_l	= modified Bessel function of first kind, order l
J_l	= Bessel function of first kind, order l
K_l	= modified Bessel function of second kind, order l
K	= absolute permeability, md [μm^2]
m	= slope of pressure, $\text{psi}/\text{day} [\text{kPa}/\text{s}]$
m	= gas cap ratio
N	= initial oil in place, STB [st m^3]
N	= Stehfest algorithm number
N_p	= cumulative oil production STB [m^3]
N_t	= total number of data points
ΔP	= pressure drop, $\text{psi} [\text{kPa}]$
q	= flow rate, $\text{bbl}/\text{d} [\text{m}^3/\text{s}]$
$Q(t_D)$	= Dimensionless water influx
$\overline{Q}(s)$	= Laplace transform of $Q(t_D)$
r_w	= reservoir radius, $\text{ft} [\text{m}]$
R_{eD}	= dimensionless aquifer radius
R_s	= gas solubility in oil, SCF/STB
s	= Laplace transform parameter
S	= storativity $h\phi C$, $\text{ft. psi}^{-1} [\text{m kPa}^{-1}]$
S_{wi}	= initial water saturation, fraction
t	= time, $\text{d} [\text{s}]$
t_D	= dimensionless time
T	= transmissibility Kh/μ , $\text{md.ft}/\text{cp} [\mu\text{m}^3/\text{Pa.s}]$
We	= water influx, $\text{Bbl} [\text{m}^3]$
W_p	= water production, $\text{Bbl} [\text{m}^3]$
Y_l	= Bessel function of second kind, order l
β_o	= oil formation volume factor, $\text{res. Bbl}/\text{STB}$
β_g	= gas formation volume factor, $\text{res. Ft}^3/\text{SCF}$
μ	= viscosity, $\text{cp} [\text{Pa.s}]$
λ	= Levenberg parameter
ϕ	= porosity, fraction

Subscripts

g	= gas
i	= initial
o	= oil
w	= water

Superscripts

—	= Laplace transform
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SI Metric Conversion Factors

bbl	x	1.589 873	E - 01 = m ³
cp	x	1.0*	E - 03 = Pa.s
ft	x	3.048*	E - 01 = m
ft ²	x	9.290 304*	E - 02 = m ²

*Conversion factor is exact.

Table 1: Data for Example 1

Values of Parameters			
N	=	120	MM STB
m	=	0.2	
R_{eD}	=	6.0	
B	=	250	Bbl/day
c	=	0.4	day⁻¹

Time (days)	Pressure psia	N _p MM STB	G.O.R SCF/STB
0	3548	0	770
105	3443	1.53	760
195	3303	4.126	746
286	3153	7.115	730
375	2938	12.00	708
470	2813	14.54	702
560	2678	18.51	740
651	2533	21.39	821
743	2453	22.77	880
835	2318	24.71	983
925	2153	26.95	1119

Table 2: Results of Regression Analysis Runs – Example 1

Run No.	Initial Guess		Final Results					SSE
	R _{eD}	C	R _{eD}	C	N	m	B	
1*			6	.4	120	.2	250	
2	8	.5	7.91	.57	106	.28	87.5	3.33 E - 5
3	6	.7	5.91	.38	121	.19	262	1.78 E - 6
4	7	.9	6.53	.17	96.95	.35	89.7	9.04 E - 5
5	5	.9	1.14	.003	93.2	.41	-ive	2.38 E - 4
6	4	.3	4.96	.628	98	.38	-ive	3.07 E - 4
7	9	.9	8.82	.97	112	.24	91.9	1.02 E - 5

* Actual values

Table 3: Results of Regression Analysis Runs – Example 2

Run No.	Initial Guess		Final Results					SSE
	R_{eD}	C	R_{eD}	C	N	m	B	
1*			5	5.67	312	0	6446	
2	4	8	3.07	9.93	240	.37	-474	5.05 E-6
3	8	4	4.81	5.04	313	-IVE	7393	3.68 E-6
4	7	9	5.25	80.3	242	.37	-125.7	5.04 E-4
5	4	3	4.01	2.83	307	-ive	1.2E4	5.0 E-6
6	6	10	6.01	9.94	321	-ive	4495	9.6 E-6
7	10	10	6.72	94	239	.38	-95	5.06 E-4

* Actual values

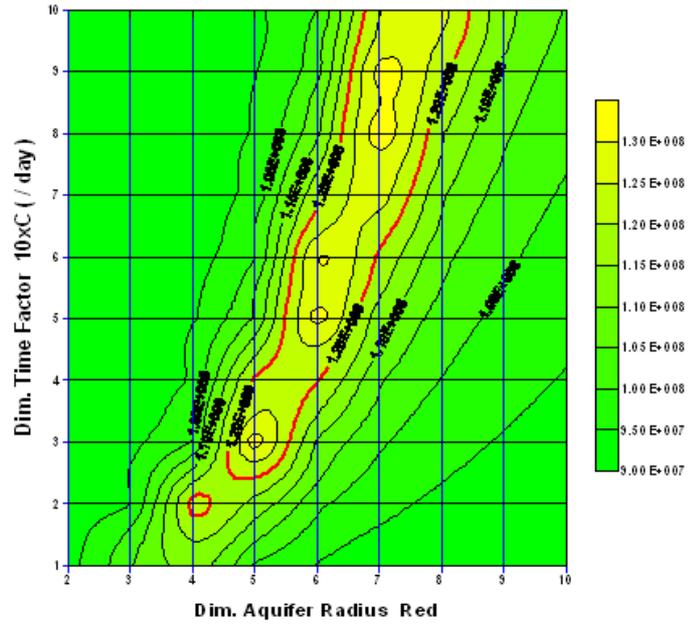


Fig. 2: Initial Oil in Place Map for Example 1

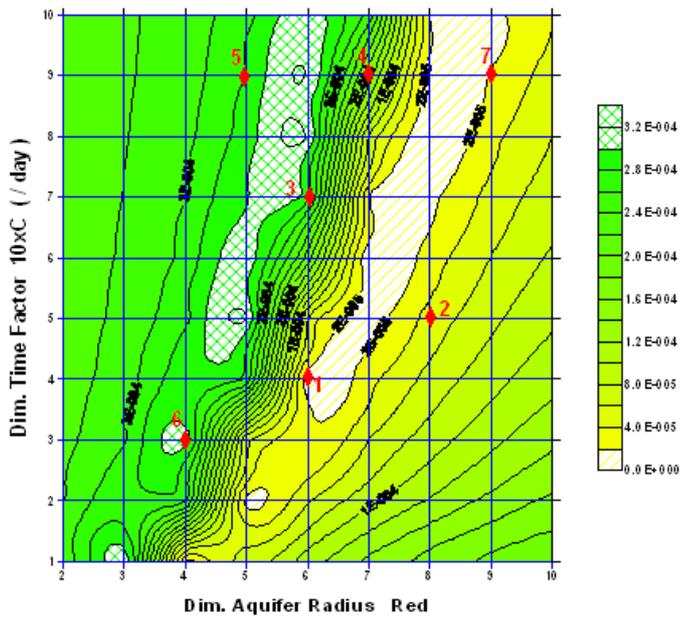


Fig. 1: Sum of Squares Map for Example 1

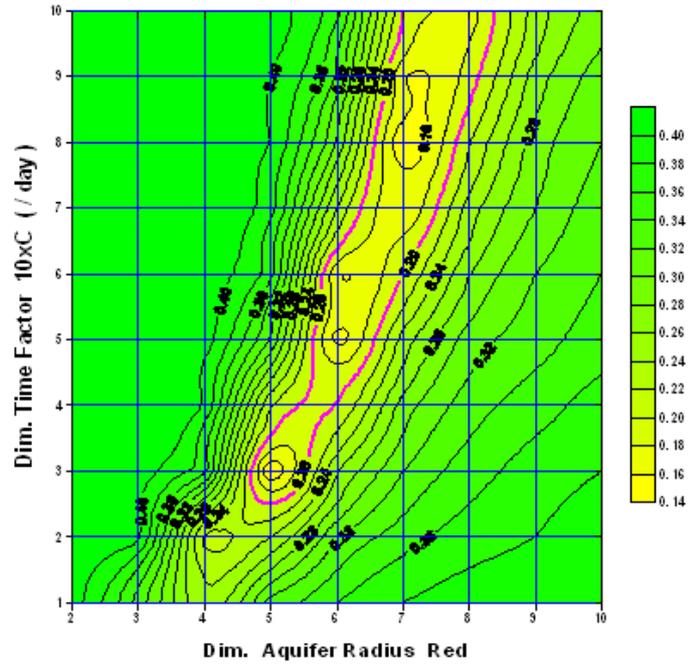


Fig. 3: Gas Cap Ratio Map for Example 1

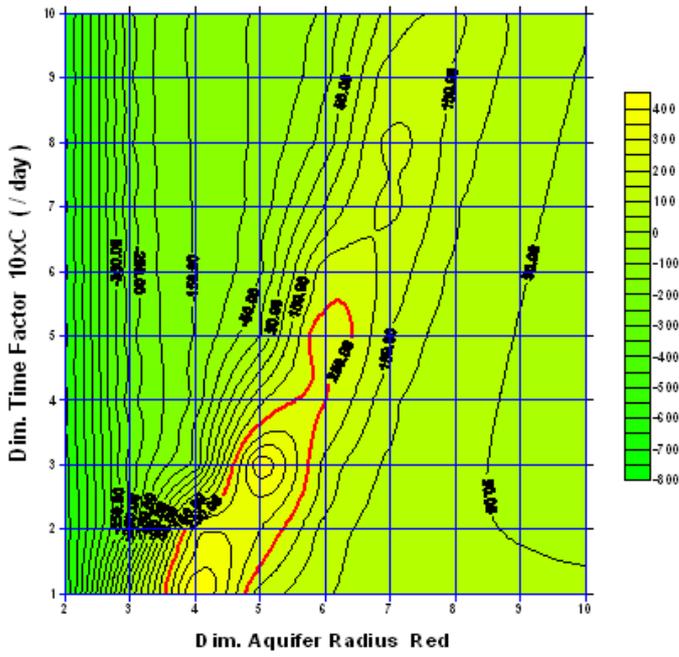


Fig. 4: Aquifer Constant B Map for Example 1

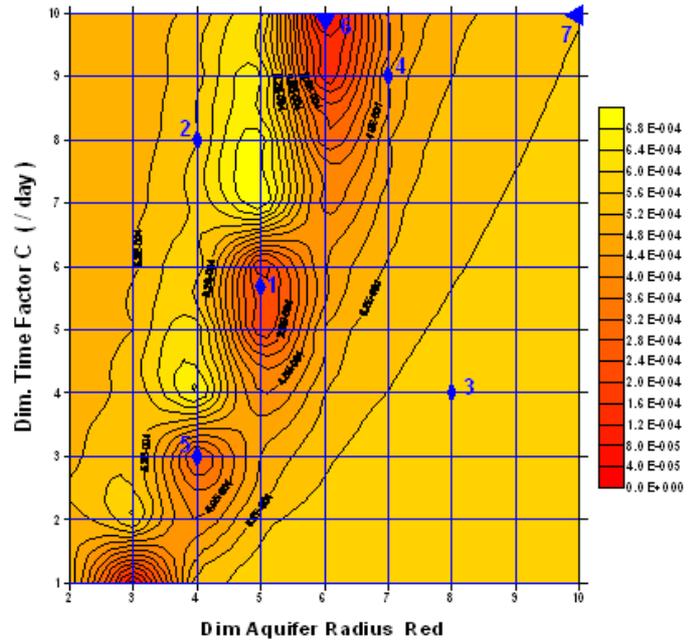


Fig. 6: Sum of Squares Map for Example 2

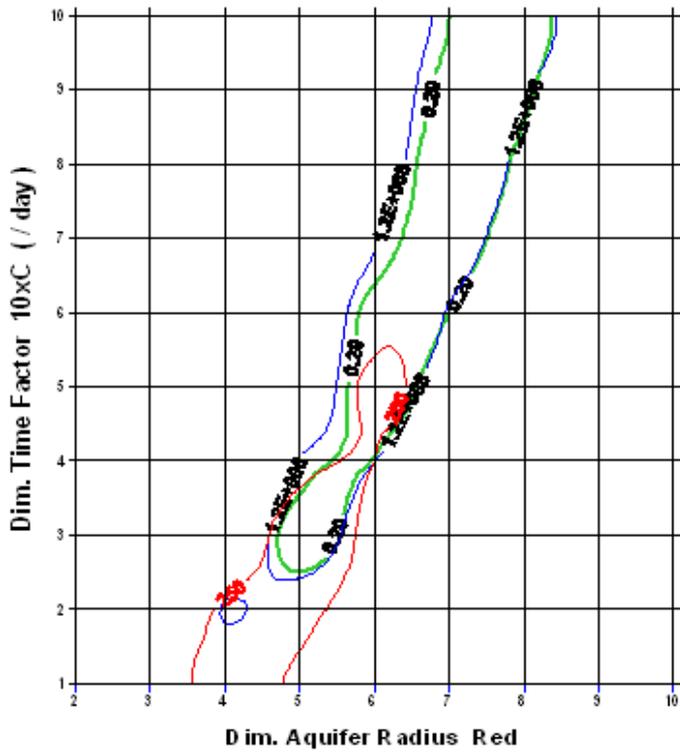


Fig.5: Overlay Map for Example 1

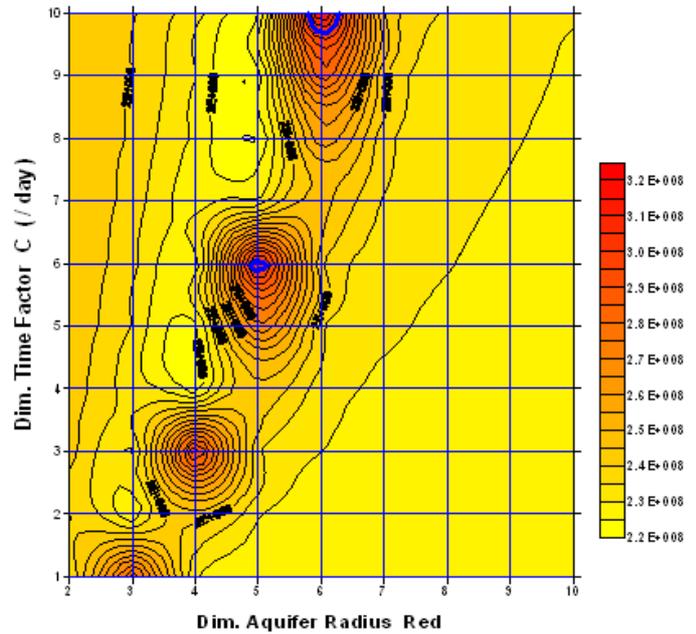


Fig. 7: Initial Oil in Place Map for Example 2

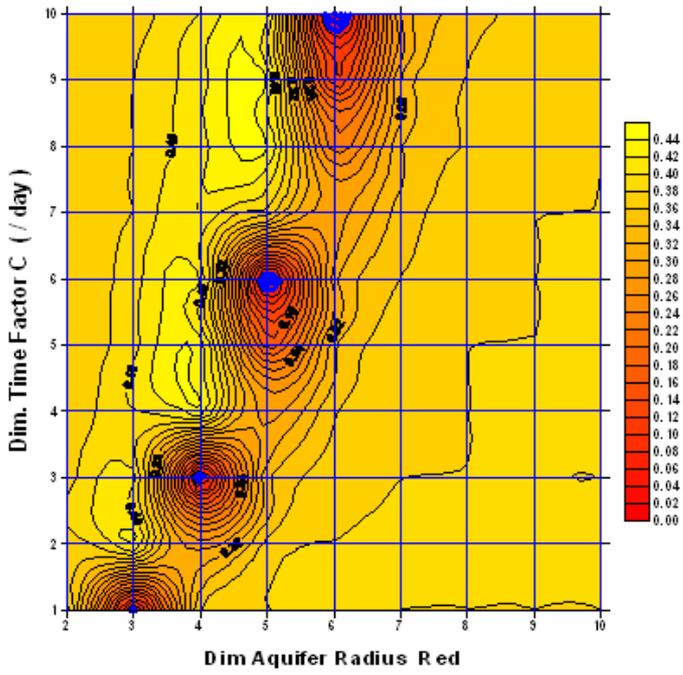


Fig. 8: Gas Cap Ratio Map for Example 2

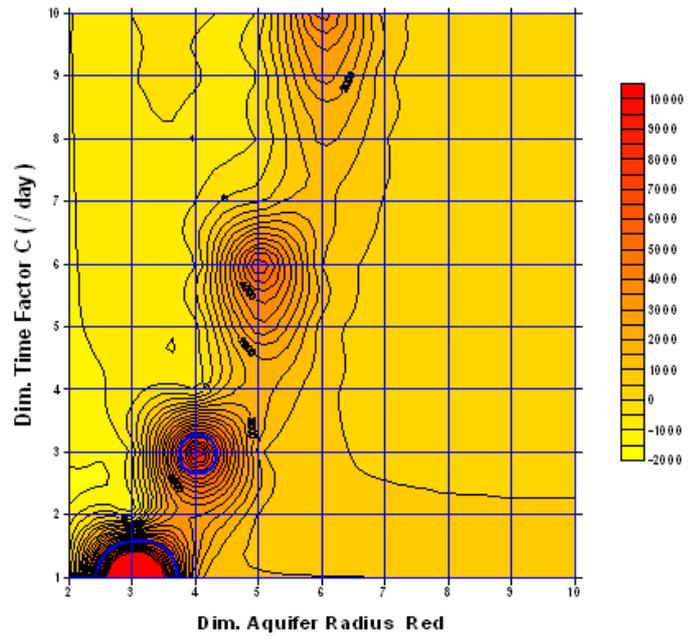


Fig. 9: Aquifer Constant B Map for Example 2