Speed Estimation for Indirect Field Oriented Control of Induction Motor Using Extended Kalman Filter

Aamir Hashim Obeid Ahmed
School of Electrical & Nuclear Engineering, Sudan University of Science and Technology (SUST)
aamirahmed@sustech.edu
Received: 11.11.2014
Accepted: 17.02.2015

ABSTRACT-Speed sensors are required for the Field Oriented Control (FOC) of induction motors. These sensors reduce the sturdiness of the system and make it expensive. Therefore, a drive system without speed sensors is required. This paper presents a detailed study of the Extended Kalman Filter (EKF) for estimating the rotor speed of an Induction Motor (IM). Using MATLAB/SIMULINK software, a simulation model is built and tested. The simulation results illustrated and demonstrated the good performance and robustness of the EKF to estimate the high and low speed. Moreover, the performance of the EKF is found to be satisfactory in case there are external disturbances.

Keywords: Induction Motor, Field Oriented Control, Sensorless, Speed Estimation, Extended Kalman Filter.

INTRODUCTION

Induction motors have been widely applied in industry because of the advantages of simple construction, ruggedness, reliability, low cost, and minimum maintenance \[1-3\]. However, to obtain fast torque response from an induction motor, the principle of field oriented control or Vector Control (VC) technique must be applied. Driven by a field oriented control, an induction motor behaves similar to a separately excited Direct Current (DC) motor in which the torque and flux are controlled separately.

Furthermore, the use of field oriented controlled induction motor drives allows obtaining several advantages compared to the DC motor in terms of robustness, size, lack of brushes, and reducing cost and maintenance \[4-8\]. There are essentially two general methods of field oriented control. One called the direct or feedback method, and the other, the indirect or feed forward method. Indirect Field Oriented Controlled (IFOC) induction motor drives are increasingly used in high performance systems due to their relative simple configuration compared to Direct Field Oriented Control (DFOC) scheme which requires flux and torque estimators. However, speed sensors are generally required for the implementation of field oriented control. The sensors include the search coils, coil taps, or Hall Effect sensors. But in most applications, speed sensor have several disadvantages, such as reduced reliability, susceptibility to noise, additional cost and weight, and increased complexity of the drive system \[4-8\].

Therefore sensorless IFOC induction motor drive eliminates the need for speed sensor, overcoming these challenges. The advantages of using sensorless IFOC induction motor drives are clear: the mechanic setup and maintenance are simpler since no shaft sensor is required, the system becomes more robust and less sensitive to the environmental noise and also the overall system cost is reduced. Like the systems using the measured speed, sensorless schemes have the disadvantage of being sensitive to motor parameter variations, especially to the rotor time constant, the
motor parameter that varies in the largest range \[^{[9-12]}\].

Various sensorless field oriented control methods for induction motor drives have been proposed using software instead of hardware speed sensor. They include different methods such as Luenberger observer (LO), Model Reference Adaptive System (MRAS), Sliding Mode Observer (SMO), Artificial Intelligence (AI), and Kalman Filer (KF). In these techniques, speed is estimated using stator voltages and currents of the induction motor and this speed is used to compare the commanded one \[^{[13-17]}\]. In this paper, the speed sensorless IFOC of induction motor drive scheme was investigated using the EKF design and simulated using MATLAB/SIMULINK software package. Simulation results are used to highlight the performance and robustness of the proposed control scheme in low and high speeds and against load torque variations.

**DYNAMIC MODEL OF INDUCTION MOTOR**

Using EKF for estimation of the rotor speed, various dynamic models are possible to be used. In order to avoid extra calculations and nonlinear transformation, stationary reference frame is preferred. The main advantages of using the model in stationary reference frame are reduced computation time, smaller sampling time, higher accuracy, more stable behaviour. A fourth order dynamic model for induction motor is developed in stationary reference frame by choosing stator currents \(i_{ds}, i_{qs}\) and rotor fluxes \(\psi_{dr}, \psi_{qr}\) as state variables and stator voltages \(v_{ds}, v_{qs}\) as input variables as follows \[^{[18]}\]:

\[
\begin{align*}
\frac{di_{ds}}{dt} &= -\alpha i_{ds} + \beta \psi_{dr} + \delta \psi_{qr} + \frac{v_{ds}}{L_a} \\
\frac{di_{qs}}{dt} &= -\alpha i_{qs} - \delta \psi_{dr} + \beta \psi_{qr} + \frac{v_{qs}}{L_a} \\
\frac{d\psi_{dr}}{dt} &= \varepsilon i_{ds} + \frac{R_s \psi_{dr}}{L_r} - \omega_r \psi_{qr} \\
\frac{d\psi_{qr}}{dt} &= \varepsilon i_{qs} + \omega_r \psi_{dr} - \frac{R_s \psi_{qr}}{L_r} 
\end{align*}
\]  

where:

\[
\begin{align*}
\alpha &= \frac{R_s + R_r L_m^2}{L_a L_r^2 L_a} \\
\beta &= \frac{R_s L_m}{L_r^3 L_a} \\
\delta &= \frac{\omega_r L_m}{L_r L_a} \\
\varepsilon &= \frac{R_r L_m}{L_r} \\
L_a &= L_s - \frac{L_m^2}{L_r}
\end{align*}
\]  

where \(L_s\) is the stator inductance, \(L_r\) is the rotor inductance, \(L_m\) is the mutual inductance, \(L_a\) is the redefined leakage inductance, \(R_s\) is the stator resistance, \(R_r\) is the rotor resistance, and \(\omega_r\) is the rotor speed.

**EXTENDED KALMAN FILTER**

The standard Kalman filter is a recursive state estimator capable of producing optimal estimates of states that are not measurable. It uses the plant's inputs and the outputs measurements, which are noisy, together with the state space model of the system. If the dynamic system of which the state is being observed is nonlinear, then the KF is called an extended one. An extended Kalman filter is a recursive optimum state observer that can be used for the state and parameter estimation of a nonlinear dynamic system in real time by using noisy monitored signals that are distributed by random noise.

This assumes that the measurement noise and system noise are uncorrelated \[^{[19-21]}\]. There are two stages to implement the extended Kalman filter algorithm. In the first stage of the calculations, the states are predicted by using a mathematical model (which contains previous estimates) and in the second stage; the predicted states are continuously corrected by using a feedback correction scheme. This scheme makes use of actual measured states, by adding a term to the predicted states (which is obtained in the first stage).

The additional term contains the weighted difference of measured and estimated output signals. Based on the deviation from the estimated value, the extended Kalman filter provides an optimum output value at the next input instant \[^{[19-21]}\].
Speed Estimation Using EKF
The main design steps for a speed sensorless induction motor drive implementation using discretized EKF technique are as follows\textsuperscript{[19-21]}:

**Step one:** Selection of the time domain induction motor model.

**Step two:** Discretization of the induction motor model.

**Step three:** Determination of the noise and state covariance matrices.

**Step four:** Implementation of the discretized EKF technique.

**Step five:** Tuning of the covariance matrices.

**Selection of the time domain IM model**
In order to estimate the rotor speed of the induction motor, the state vector must be extended to include rotor speed. The extended induction motor model can be expressed as follows\textsuperscript{[19-21]}:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) = h(x)
\end{align*}
\]

where:

\[
A = \begin{bmatrix}
-\alpha & 0 & \beta & \delta & 0 \\ 0 & -\alpha & -\delta & \beta & 0 \\ \varepsilon & 0 & -\frac{R_r}{L_r} & -\omega_r & 0 \\ 0 & \varepsilon & \omega_r & -\frac{R_r}{L_r} & 0 \\ 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
\frac{1}{L_a} \\ 0 \\ \frac{1}{L_a} \\ 0 \\ 0
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\]

\[
h(x) = \begin{bmatrix}
i_{ds} \\ i_{qs}
\end{bmatrix}
\]

**Discretization of the induction motor model**
For digital implementation of the EKF, the discretized induction motor equations are required. The time discrete state space model of the induction motor can be obtained from Equation (3) as follow\textsuperscript{[19-21]}:

\[
\begin{align*}
x(k+1) &= \Gamma x(K) + Gu(k) \\
x(k+1) &= f(x(k), u(k)) \\
y(k) &= Hx(k)
\end{align*}
\]

where $\Gamma$, $G$, and $H$ are discretized system matrix, input matrix, and output matrix, respectively. The discretized matrices are derived using the exponential Taylor approximation, assuming a small sampling time and the use of Zero-Order-Hold (ZOH) sampling technique. They are:

\[
\Gamma = I + AT_s
\]

\[
G = BT_s = \begin{bmatrix}
T_s & 0 \\ 0 & \frac{T_s}{L_a} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0
\end{bmatrix}
\]

\[
f(x(k), u(k)) = \begin{bmatrix}
\chi i_{ds}(k) + T_s \beta \psi_{dr}(k) + T_s \Phi \psi_{qr}(k) + \eta \\
\chi i_{qs}(k) - T_s \Phi \psi_{dr}(k) + T_s \beta \psi_{qr}(k) + \Delta \\
T_s \varepsilon i_{ds}(k) + \rho \psi_{dr}(k) - T_s \omega_r(k) \psi_{qr}(k) \\
T_s \varepsilon i_{qs}(k) + T_s \omega_r(k) \psi_{dr}(k) + \rho \psi_{qr}(k) \\
\omega_r(k)
\end{bmatrix}
\]
where:

\[ \rho = 1 - T_s \frac{R_r}{L_r} \]

\[ \chi = 1 - T_s \alpha \]

\[ \Phi = \frac{\omega_x(k)L_m}{L_r L_a} \]  \hspace{1cm} (7)

\[ \eta = \frac{T_s v_{q0}(k)}{L_a} \]

\[ \Delta = \frac{T_s v_{q1}(k)}{L_a} \]

\( T_s \) denotes the sampling time and \( I \) is an identity matrix. Unlike a deterministic system where a plant is assumed to be perfect, a stochastic system is used to represent a state space model of a system with the presence of disturbance which may due to simplification of the modeling, unmeasured input disturbances and also measurement noise from the sensor. The influence of these disturbances or noises on the system in the discrete state space model is shown below:

\[ x(k+1) = f(x(k),u(k)) + w(k) \]

\[ y(k) = Hx(k) + v(k) \]  \hspace{1cm} (8)

where \( w(k) \) and \( v(k) \) are characterized as zero mean, white Gaussian noise and having zero cross correlation with each other \(^{[19-21]}\).

**Determination of the Q, R, and P matrices**

The purpose of the KF is to obtain the unmeasurable states by using the measured states and also the statistics of the noise and measurements. In general, the computational inaccuracies, modeling error and errors in the measurements are considered by means of noise inputs. A critical part of the design of the EKF is to use correct initial values for the various covariance matrices. These have important effects on the filter stability and convergence time. To obtain the best estimate value of the speed, it is important to use accurate initial values for the covariance matrices of the system noise Q, measurement noise R and the state noise P. The elements of Q and R depend on the number of state variables. The system noise matrix is 5×5 matrix and the measurement noise matrix is 2×2 matrix. This should require the knowledge of 29 elements. However, by assuming that the noise signals are not correlated, both Q and R are diagonal and only five elements must be known in Q and two elements in R. Generally, the parameters in the direct and quadrature axes are same. This implies that the first two elements in the diagonal of Q are equal and the third and fourth elements are also equal. So \( Q = \text{diag}(q_{11}, q_{11}, q_{33}, q_{33}, q_{55}) \) contains only three elements. Similarly, the two diagonal elements in the measurement noise matrix are equal, thus \( R = \text{diag}(r, r) \). It follows that in total only four noise covariance elements must be known. The initial values of the covariance matrices of the system noise and measurement noise are selected randomly and tuned accordingly \(^{[19-21]}\).

**Implementation of the discretized EKF**

Using of the extended Kalman filter includes two stages: the prediction and correction (filtering) stages. In prediction stage, state prediction values and state error covariance predictive value is calculated, while in filtering stage, the gain of Kalman filter is calculated and the state error covariance predicative value is updated. The steps to use extended Kalman filter to estimate the induction motor speed include \(^{[21]}\):  

**Step one:** Initial the state error covariance matrix \( P(0) \) and the initial state \( x(0) \).

**Step two:** Set the system noise covariance matrix as \( Q(5\times5) \) and the measurement noise covariance matrix as \( R(2\times2) \).

**Step three:** in each sampling period carry out the following extended Kalman filter iteration.

(a) **Prediction of the state vector.**

\[ \hat{x}(k+1) = f\left(\hat{x}(k),u(k)\right) \]  \hspace{1cm} (9)

where \( \hat{x} \) denotes the estimation value.

(b) **Estimate the error covariance matrix.**

\[ P(k+1) = F(k)P(k/k)F(k)^T + Q \]  \hspace{1cm} (10)

Where \( F(k) \) is the Jacobin matrix of partial derivatives of \( f(x(k),u(k)) \) with respect to \( x(k) \), defined as:


\[ F(k) = \frac{\partial f(x(k), u(k))}{\partial x(k)} \bigg|_{x(k)=\hat{x}(k/k)} \]

\[ F(k) = \begin{bmatrix} \chi & 0 & T_s \beta & T_s \Phi & \theta \\ 0 & \chi & -T_s \Phi & T_s \beta & \lambda \\ 0 & T_s \varepsilon & \rho & \vartheta & \zeta \\ 0 & T_s \varepsilon & T_s \omega_r(k) & \rho & \ell \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \]

where:

\[ \theta = \frac{T_s L_m \psi_{qr}(k)}{L_r L_d} \]
\[ \lambda = \frac{T_s L_m \psi_{dr}(k)}{L_r L_d} \]
\[ \zeta = -T_s \psi_{qr}(k) \]
\[ \ell = T_s \psi_{dr}(k) \]
\[ \vartheta = -T_s \omega_r(k) \]

(c) Calculate the Kalman gain.

\[ K(k+1) = P(k+1/k) H^T + \begin{bmatrix} HP(k+1/k) H^T + R \end{bmatrix}^{-1} \]

(d) Update state vector.

\[ \hat{x}(k + 1/k + 1) = \hat{x}(k + 1/k) + K(k+1) [y(k+1) - H\hat{x}(k+1/k)] \]

(e) Update the error covariance matrix.

\[ P(k+1/k+1) = P(k+1/k) - K(k+1) H P(k+1/k) \]

RESULTS AND DISCUSSION

The simulations are done for the rotor speed estimation of IM by indirect field oriented control technique with EKF method using the MATLAB/SIMULINK software package. The MATLAB/SIMULINK block diagram of speed sensorless control of indirect field oriented controlled IM using EKF shown in Figure 1. The IM parameters used in simulations are given in Table I.
Table I: Parameters of the induction motor

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor resistance, $R_r$</td>
<td>0.72 Ω</td>
</tr>
<tr>
<td>Stator resistance, $R_s$</td>
<td>0.55 Ω</td>
</tr>
<tr>
<td>Rotor inductance, $L_r$</td>
<td>0.068 H</td>
</tr>
<tr>
<td>Stator inductance, $L_s$</td>
<td>0.068 H</td>
</tr>
<tr>
<td>Magnetizing inductance, $L_m$</td>
<td>0.063 H</td>
</tr>
<tr>
<td>Moment of inertia, $J$</td>
<td>0.05 kg.m²</td>
</tr>
<tr>
<td>Viscous friction coefficient, $B$</td>
<td>0.002 Nms⁻¹</td>
</tr>
</tbody>
</table>

In order to show the performances and the robustness of the EKF technique, we simulated different operating cases which are presented thereafter. The sampling time used in the simulation is 0.01sec. In extended Kalman filter, matrixes $Q$ and $R$ are difficult to be known exactly because the disturbances $w$ and $v$ are not known. The method to determine the $Q$ and $R$ matrices are trial-error based. The initial values of the $Q$ and $R$ matrices are set as:

$$Q = \begin{bmatrix}
10^8 & 0 & 0 & 0 & 0 \\
0 & 10^8 & 0 & 0 & 0 \\
0 & 0 & 10^9 & 0 & 0 \\
0 & 0 & 0 & 10^9 & 0 \\
0 & 0 & 0 & 0 & 10^8
\end{bmatrix}$$

(16)

$$R = \begin{bmatrix}
10^2 & 0 \\
0 & 10^2
\end{bmatrix}$$

Case one: Constant speed
To test the performance of the speed sensorless induction motor drive at a constant speed without loadtorque. The induction motor was allowed to accelerate from zero to 70 rad/sec. The simulation ran for 5 seconds. The actual (real) speed, estimated speed and command (reference) speed are plotted in Figure 2.

The estimated speed, real speed and command speed are plotted with respect to time on the same scale to observe the accuracy of extended Kalman filter speed estimator. The steady state was reached at 0.44 seconds. It can be seen that there is a very good accordance between real speed and estimated speed without overshoot and any steady state error.

Case two: Variable speed
Figure 3 shows the behavior of induction motor speed estimation under variable command speed with no load torque, where the command speed is first set at 70 rad/sec, at 2 seconds the reference speed is changed to 100 rad/sec, at 4 seconds the reference speed is changed to 0 rad/sec (zero speed), finally at 6 seconds the command speed is changed to 50 rad/sec.

This result shows clearly a very satisfactory performances in tracking, the measured or actual induction motor speed perfectly follows the reference trajectory, with a minimal tracking error.

The observer’s response illustrates an excellent precision of the estimated speed for high and low speeds, but also at zero speed operating.

Case three: Inversion of the speed
To test the robustness of the sensorless control system, we applied a changing of the speed reference from 100 rad/sec to -100 rad/sec under no load torque. Figure 4 presents the estimated speed, actual speed, and reference speed.

In both forward and reverse directions the estimated speed tracks the measured speed with good agreement with no steady state error. The EKFestimation technique is robust because the variation of the speed is important and the estimated speed follows the actual speed when the induction motor starts and at the moment of speed inversion.

![Figure 2: Simulation result at constant speed](image-url)
Case four: Load torque
In order to testify the robustness of the controlled system, a 1Nm load torque is suddenly added at time 2 seconds and then removed at time 3 seconds while the reference speed is set as 70rad/sec. Figure 5 shows the speed sensorless control performance where the load torque was applied and rejected. It is seen that the estimated and actual speeds can track the trajectory of the reference speed very well. However, a small dip occurs in the estimated and actual speeds at the instant of load torque.

CONCLUSIONS
In this paper, an extended Kalman filter technique for indirect field oriented control of the induction motor drive has been proposed and tested. In this proposed scheme, the estimate of rotor speed is obtained from the directly measurable stator currents and voltages. For this purpose, appropriate mathematical model of induction motor is studied and discretized for real time applications. The effectiveness of the proposed technique was confirmed through MATLAB/SIMULINK simulation results in different induction motor operating conditions. Simulation results show good performance and robustness of the speed sensorless indirect field oriented control of the induction motor drive at high and low speeds. In addition, the simulation results show the robustness of the proposed algorithm against load torque variations.

REFERENCES


