Analysis of Shear Wall Structures using the Moment Transformation Method

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Abstract: This paper presents a simplified method of analysis "The Moment Transformation Method". The method is suitable for the analysis of tall buildings including shear walls subjected to both vertical and horizontal loads. In this method, the concept of the "Carry-over moments matrix" is introduced. The moments are transformed in groups from one level to the subsequent level. The final rotations and moments in each level are then calculated. Hence, the shear forces and lateral displacements are obtained from the rotations and moments. A computer program "MTProg" was developed and implemented for the method. Problems ranging from the simple planar frame-shear wall interaction up to the complex three dimensional shear wall structures with random arrangements and orientation of components and subjected to vertical and horizontal loads are studied. The accuracy of the results obtained is verified by comparison with known results and with those obtained using STAADPro and ETABS. The comparison shows, clearly, that the results are in good agreement, thus verifying the accuracy of the proposed method.

Keywords: Tall buildings, Shear walls, Moment Transformation, Equivalent Stiffness

Introduction:
Simplified methods of analysis of tall buildings are required to minimize the analysis cost especially in the preliminary design stage when the analysis is carried out and modified several times before the final design. The available analysis methods are of three types: (a) Iterative methods (b) simplified matrix methods (c) differential equation approaches and continuum methods. There are a lot of developments in the proposed methods of solution [10],[11],[12].

As stated by Ghali and Neville [1], Clough, King, and Wilson developed a simplified matrix method for analyzing frames with or without shear walls included in the plan (two dimensional analysis. The method results in a lateral stiffness matrix with only one degree of freedom per floor. They also, refer to the Substitute-Frame method derived by Lightfoot 1] to simplify the problem. The method results in a structure composed of two systems connected by a rigid inextensible link of members at all the floor levels, and the axial deformation is neglected. As also stated in [1] an iteration method for calculating the side-way of the Frame-Shear wall system has been suggested by Khan and Sbarounis, who give charts to assist in practical design. A three dimensional analysis of shear walls structures is proposed by Ghali and Neville [1] wherein the degrees of freedom are reduced to three per floor. The resulting degrees of freedom for each principal axis of each shear wall will be twice the number of floors (rotations and translations DOFs) are condensed to translation only. All the developed transformed matrices are superimposed to produce a global stiffness matrix for the whole building. Using the external loads applied at the assumed
Jaeger, Mufti & Mamet [2] proposed an analytical theory for the analysis of tall three dimensional multiple shear wall buildings. The results they obtained are compared with data obtained by the finite element method and experiments conducted on a seven storey multiple shear wall model.

A two-level finite element technique of constructing a frame super-element was created by Leung and Cheung [3] to reduce the computational effort for solving large scale frame problems. The order of the overall matrices is greatly reduced by considering the frame as a super-element connected to other elements by means of master nodes. The accuracy of solution is improved either with finer subdivision or by taking more master nodes inside each super element.

Leung [4] developed a rational method, for the analysis of plane frames. The method is based on the concept of distribution factors which are allowed to vary from floor to floor and are determined by using three floors at a time. Thus, the number of degrees of freedom is reduced to three at any one floor. The method [4] was generalized to three dimensional frames by Leung [5]. Linear combinations of distribution factors with mixing factors as weighting factors give the actual displacements at the nodes. Structural idealizations of coupled shear walls by beams and columns were recommended. In order to improve the results another three additional sets of global distribution factors were introduced by Leung and Wong [6] to account for the uneven elongation (shortening) of the columns having unevenly distributed stiffness along the height and across the floor plane. The total number of unknowns per floor is reduced. Using the concept of the two-level finite-element method, the global distribution factors of the building frame were obtained. The global and local distribution factors together predict lateral and torsion deflections and internal nodal displacements accurately. It had been demonstrated that this method is both fast and economical.

Wong and Lau [7] presented a simplified finite element for analysis of tall buildings. It is based on the assumption that the warping displacement modes of a floor and the differences between neighboring floors are mainly determined by the local structural characteristics. Once the warping modes are determined, these modes are taken as the basis of generalized coordinates. Then, the problem can be reduced to a formulation in which only the rigid body displacements and the warping generalized coordinates of each floor are unknown. They state that: when suitable warping modes from a multi-storey sub-model are chosen, the proposed simplified finite element method is inexpensive and is able to yield sufficiently good results for practical design purposes.

An efficient finite strip analysis of Frame-Shear wall tall building was prepared by Swaddiwudhipong, Lim and Lee [8], for the analysis of coupled frame-shear wall buildings subjected to lateral loads. They claim that the method provides reasonably accurate results, requires a small core storage and short computing time and is suitable for implementation on any of the personal computers which are commonly available in most engineering design offices.

A simplified approach for seismic calculation of a tall building braced by shear walls and thin-walled open section structures was presented by Meftah, Tounsi and El Abbas [9]. An approximate hand-method for seismic analysis of an asymmetric building structure having constant properties along its height was presented. The building was stiffened by a combination of shear walls and thin-walled open section structures. The proposed method results were compared with finite element calculations.

The moment distribution method was primarily designed for frames without side-sway. But, Hardy Cross [17], showed that the side-sway in multi-storey buildings can be handled by giving each storey a horizontal unit displacement and then calculating the shear
forces obtained in all stories. By combining the solutions, the true shear and moments can be calculated in all stories. Grinter\cite{17}, proposed a method for multi-storey frames with side-sway which he called the method of 'successive corrections' based on the Hardy Cross method. Instead of the sway correction method, a new kind of moment distribution was presented and used to reduce the solution of frames subjected to lateral loads but it was tailored for substitute frames composed of single column.

The method was called 'The no-shear moment distribution' and sometimes also known as 'The cantilever moment distribution'\cite{1}. The no-shear moment distribution was based on the concept of distribution of the sway fixed end moments without changing the sway-moment equation during the distribution procedure.

The concept of the direct moment distribution was suggested by Lin\cite{16} as a means of eliminating the iteration required in the standard moment distribution procedure. Other alternative methods e.g. the precise moment distribution had been developed for the direct distribution of moments\cite{13}.

In this paper, a new simplified method of analysis of two and three dimensional buildings "The Moment Transformation Method" is presented. A computer program "MTProg" is developed for the method. The method is based on the concept of the direct moment distribution.

If $m$ is the number of degrees of freedom in one floor, corresponding to the columns and the shear walls in a building of total number of floors $N$, by using the proposed simplified method the solution for the large number of $N*m$ unknowns is reduced to $N$ solutions of a small number of $m$ unknowns each, and a large amount of computing efforts can be saved.

The transformation method can be used to simplify the analysis of the continuous beams and the multi-bay sub frames composed of one level and connected by a single or double column at the joints same as the direct moment distribution procedure but with different formulation. The method can also be used to solve the problems of the single post connected by horizontal members and subjected to lateral forces, and permitted to sway freely based on the concept of the no-shear moment distribution. The method has also been developed and generalized to solve the more complex two dimensional and three dimensional multi-floors structural systems with irregular arrangement and orientation of the columns and the shear walls and subjected to both vertical and horizontal loads.

The Moment Transformation Method

Two-member Plane Frame

The main requirement is to find an equivalent member which can replace the two members connected as shown in Figure 1.

![Figure 1. Two Members Frame (a) System #1 (b) System #2](image)

**The Remaining Frame**

Figure 1. Two Members Frame (a) System #1 (b) System #2

Assume that:

- $M_A$: is moment applied at joint #1.
- $\theta_1$: rotation angle in radians at joint #1.
- $\theta_2$: rotation angle in radians at joint #2.

$M$: is equivalent moment at joint #2.

By applying the slope deflection method for system#2, shown in the Figure 1(b), at joint # 1:

$$(S_1 + S_2) \cdot \theta_1 + t_2 \cdot \theta_2 = 0$$  \hspace{1cm} (1)

From which:

$$\theta_1 = \left[ - \frac{t_2}{(S_1 + S_2)}\right] \cdot \theta_2$$  \hspace{1cm} (2)

and at joint # 2:
\[ M = S_2 \cdot 02 + t_2 \cdot 01 \]  \hspace{1cm} (3)

Substitution for \( \theta 1 \) from (2) into equation (3), yields:
\[ M = S_2 \cdot 02 - t_2^2 / (S_1 + S_2) \cdot 02 \]

From Figure 2, \( M = S \cdot 02 \). Therefore:
\[ M = S_e \cdot 02 \]

Applying the reciprocal theorem for the two systems #1 and #2, and since the configurations of the two systems are identical, therefore:
\[ M \cdot 02 = M_A \cdot 01 \]

and substitute for \( \theta 1 \) from equation (3):
\[ M = M_A [ -t_2 / (S_1 + S_2)] \]

where:
\[ TF = -t_2 / (S_1 + S_2) \]  \hspace{1cm} (5)

From equations (4) and (5), \( S_e \) can also be expressed as:
\[ S_e = S_2 + TF \cdot t_2 \]

where:
\( S_e \): is the equivalent stiffness of the members #1 and #2, or in other words, is the stiffness of the member equivalent to members #1 and #2. Figure 2.

\( TF \): is the transformation factor used to transform the moment \( M_A \) from joint #1 towards joint #2.

\( S_i \): is the ordinary rotation stiffness of member #i \((S_i = 4EI/L, \) considering bending deformation only, and \( S_i = [(4+\alpha)/(1+\alpha)](EI/L)_i, \) considering both bending and shear deformations).

\( t_i \): is the ordinary carry-over moment of member #i \((t_i = 2EI/L, \) Considering bending deformation only, and \( t_i = [(2-\alpha)/(1+\alpha)](EI/L)_i, \) considering both bending and shear deformations).

\( a = [(12EI)/(G.a_r.L^2)]_i \)

\( E \): Modulus of Elasticity.

\( G \): Modulus of torsional rigidity.

\( L \): Length of member #i.

\( I \): Moment of inertia of the section.

\( a_r \): Shear area of the section.

In the special case of prismatic members neglecting shear deformation:

The equivalent stiffness of the two members 1 and 2, becomes:
\[ Ke = [(0.75+K_i/K_2)/(1.00+K_i/K_2)]K_2 \]  \hspace{1cm} (6)

And the moment transformation factor, becomes:
\[ TF = -K_2 / [2*(K_1+K_2)] \]  \hspace{1cm} (7)

where \( K_i = (4EI/L)_i \)

**Multi-Bay, Multi-Storey Building**

The transformation is from joint to joint through beams in the case of continuous beam or multi-bay single storey frame, Figure 3, or through columns in single post subjected to lateral forces, Figure 4a. The system now is composed of multi-bay multi floors and the transformation will be carried out through the floors from top to bottom and from bottom to top. Hence, the transformation now will be carried from one level to another level, as shown in Figure b. The transformation procedure can be generalized to obtain the equivalent stiffness matrix and moment transformation factors matrix as follows: The Moments Transformation procedure from top to bottom levels, gives:

\[ [SR] = [NN]_i, \text{ (if } i = 1) \]  \hspace{1cm} (8)

\[ [SR] = [NN]_i + [GG]_{i-1}, \text{ (if } i \neq 1) \]  \hspace{1cm} (9)

\[ [AA] = [A]_i + [SR] \]  \hspace{1cm} (10)

\[ [FF]_i = -[B]_i^T [AA]_i \]  \hspace{1cm} (11)

\[ [GG]_i = [A]_i + [FF]_i [ B ]_i \]  \hspace{1cm} (12)

where:

\[ [NN]_i \] is the Over all Rotation Stiffness Matrix of the Level # i. \([GG]_{i-1} \] , is the Equivalent Rotation Stiffness Matrix of The Floor # i-1. \([SR] \) , is the summation of \([NN]_i \) and \([GG]_{i-1} \).
Figure 3: Continuous Frame With One Floor Level

Figure 4: (a) Frame With Single Post. (b) Multi-Storey Two Or Three Dim. Building

\[ [AA] \text{, is the summation of } [A]_i \text{ and } [SR]. \]
\[ [FF]_i \text{, is the Transformation Factors Matrix of The Floor } # i. \]
\[ [B]_i \text{, is the Carry-Over Moments Matrix of The Floor } # i. \]
\[ [GG]_i \text{, is the Equivalent Rotation Stiffness Matrix of The Floor } # i. \]

The same procedure can be used for transformation from bottom to top.

Condensed Stiffness and Carry-Over Moments Matrices for a Single Post:
A single post subjected to side sway as a result of application of lateral loads with no shear produced can be analyzed as follows:

The corresponding stiffness matrix equation for the two DOFs is:

Figure 5: Rotation And Translation DOFs of a Single Post.
\[
\begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
D_1 \\
D_2
\end{pmatrix}
= \begin{pmatrix}
F_1 \\
F_2
\end{pmatrix}
\] (13)

In order to obtain the condensed stiffness matrix, make: \(F_1 = S^*\), \(F_2 = 0\), \(D_1 = I\) and, \(D_2 = D\).

where \(S^*\) is the modified stiffness or rotation stiffness with translation not prevented.

Solving for \(D\) and \(S^*\) after substituting the values above in (12) gives:

\[
D = - S_{22}^{-1} \cdot S_{21}
\] (14)

\[
S^* = S_{11} - S_{12} \cdot S_{22}^{-1} \cdot S_{21}
\] (15)

Considered bending deformation only, and neglecting shear deformations, the displacement \(D\) and the modified stiffness \(S^*\) are:

\[
D = - 0.5 L
\]

\[
S^* = EI/L
\]

And, for equilibrium of the post:

\[
t^* = - S^*
\] (16)

where: \(t^*\) is the carry-over moment corresponding to \(S^*\).

The lateral loads are applied in terms of side sway fixed moments with no rotation of the joints permitted.

Condensed Stiffness and Carry-Over moment matrices for Multiple Columns and Shear Walls:

The procedure used to obtain the condensed stiffness for a single member subjected to side sway is now generalized to a bundle of members. Considering a system of two vertical members (columns or walls), the stiffness matrix equation corresponding to the three degrees of freedom shown in Figure 7 will be as follows:

The procedure used to obtain the condensed stiffness for a single member subjected to side sway is now generalized to a bundle of members. Considering a system of two vertical members (columns or walls), the stiffness matrix equation corresponding to the three degrees of freedom shown in Figure 7.

Condensation of the total matrix to only rotations, Figure 8, yields:

\[
[D_1 \ D_2] = \begin{pmatrix}
(S_{31} / S_{33}) \\
(S_{32} / S_{33}) \\
\end{pmatrix}
\]

\[
[0 \ 0]
\]

(18)

And;

\[
\begin{pmatrix}
S_{11}^* & S_{12}^* \\
S_{21}^* & S_{22}^*
\end{pmatrix}
= \begin{pmatrix}
S_{11} - S_{12} \cdot S_{33}^{-1} \cdot S_{31} \\
S_{22} - S_{23} \cdot S_{33}^{-1} \cdot S_{32}
\end{pmatrix}
\]

The internal interaction force, \((F_i)_{ij}\) is obtained from the different stiffness configurations and hence the elements of the carry-over moments matrix are calculated from the following equation:

\[
t^*_{ij} = - S^*_{ij} + (F_i)_{ij}^* \cdot L
\] (19)
Three dimensional analysis of buildings composed of randomly arranged and oriented different Shear Walls:
The analysis of the three dimensional buildings is the same as that of the two dimensional one. But, the lateral displacements of the three dimensional systems are expected to be in all possible directions in the floor level, and the floors also may be subjected to twist rotations. Instead of the inextensible truss member used previously in the two dimensional systems, the rigid in-plane floor will be used here, with the out of plane stiffness not considered in this stage of the analysis. The carry-over moment matrix is constructed following a procedure similar to that for the two dimensional analysis.

**Numerical Examples**
Using the computerized proposed simplified method, five examples representing different shapes and arrangements were studied. A sixth example was used to compare computer storage and running time, as follows:

**Two Floor One Bay Portal Frames:**
The bending moments are obtained using the simplified method for a two storey frame under the vertical and horizontal loading shown in the Figure 9. The relative values of $K = I/L$ are indicated in Figure 9.

The bending moments results obtained are compared with the results from reference $[1]$ and are shown in Tables 1 (a&b) and Figure 10.

**Table 1a. Comparison of bending moments (M x $q b^2/100$).**

<table>
<thead>
<tr>
<th>Results</th>
<th>AB</th>
<th>BA</th>
<th>BE</th>
<th>BC</th>
<th>CB</th>
<th>CD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ghali &amp; Neville</td>
<td>8.7</td>
<td>-12.3</td>
<td>34.2</td>
<td>-21.9</td>
<td>-21.5</td>
<td>21.5</td>
</tr>
<tr>
<td>%Difference</td>
<td>3.816</td>
<td>3.447</td>
<td>0.538</td>
<td>-1.096</td>
<td>1.047</td>
<td>1.047</td>
</tr>
</tbody>
</table>

**Table 1b. Comparison of bending moments (M x $q b^2/100$).**

<table>
<thead>
<tr>
<th>Results</th>
<th>FE</th>
<th>EF</th>
<th>EB</th>
<th>ED</th>
<th>DE</th>
<th>DC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ghali &amp; Neville</td>
<td>43.1</td>
<td>56.5</td>
<td>-88.5</td>
<td>32.3</td>
<td>35.1</td>
<td>-35.1</td>
</tr>
<tr>
<td>MTProg</td>
<td>43.494</td>
<td>56.199</td>
<td>-88.693</td>
<td>32.494</td>
<td>34.891</td>
<td>-34.891</td>
</tr>
<tr>
<td>%Difference</td>
<td>0.914</td>
<td>-0.533</td>
<td>0.218</td>
<td>0.601</td>
<td>-0.595</td>
<td>-0.595</td>
</tr>
</tbody>
</table>
Figure 10. Comparison of Bending-moment diagram for two floor-one bay portal Frame (M x qb²/100)

The reference results are approximate values, calculated by using the moment distribution method. The largest percentage difference (3.816 %) is in the small values of the bending moment (9.032) and is on the safe side. The other values are almost the same. The bending-moments diagrams are shown in Figure 10, showing very close agreement.

Three Dimensional Multi-bay four storey building:

The approximate values of the end-moments in a column and a shear wall in a structure that has the plan shown in figure 11 are found using the simplified method. The structure has four stories of equal heights h = b. The frame is subjected to a horizontal force in the x direction of magnitude P/2 at the top floor and P at each of the other floor levels. The properties of members are as follows: for the columns I = 17×10⁻⁶ b⁴, for beams I = 34×10⁻⁶ b⁴, and for walls I = 87 ×10⁻³ b⁴, Young modulus, E = 2.3 G. The area of wall cross section = 222 × 10⁻³ b². The shear deformation is considered in the walls only.

In the actual structure there are 16 columns, 4 walls, 12 beams of length 1.6 b, and 4 beams of length 2 b.

Comparisons of the results from the program MTProg and reference \(^{(1)}\) are shown in tables 2, 3 and 4 and Figures 13 and 14.

<table>
<thead>
<tr>
<th>Floor #</th>
<th>Ghali &amp; Neville</th>
<th>MTProg</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.47</td>
<td>11.45</td>
<td>-0.174</td>
</tr>
<tr>
<td>2</td>
<td>8.32</td>
<td>8.31</td>
<td>-0.120</td>
</tr>
<tr>
<td>3</td>
<td>5.03</td>
<td>5.03</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>2.03</td>
<td>2.03</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Floor #</th>
<th>Ghali &amp; Neville</th>
<th>MTProg</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4368 P</td>
<td>0.4365 P</td>
<td>-0.069</td>
</tr>
<tr>
<td>2</td>
<td>1.4418 P</td>
<td>1.4415 P</td>
<td>-0.021</td>
</tr>
<tr>
<td>3</td>
<td>2.4459 P</td>
<td>2.4456 P</td>
<td>-0.012</td>
</tr>
<tr>
<td>4</td>
<td>3.4542 P</td>
<td>3.4539 P</td>
<td>-0.009</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Floor #</th>
<th>Reference</th>
<th>MTProg</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0632 P</td>
<td>0.0635 P</td>
<td>0.475</td>
</tr>
<tr>
<td>2</td>
<td>0.0580 P</td>
<td>0.0585 P</td>
<td>0.862</td>
</tr>
<tr>
<td>3</td>
<td>0.0540 P</td>
<td>0.0544 P</td>
<td>0.741</td>
</tr>
<tr>
<td>4</td>
<td>0.0460 P</td>
<td>0.0461 P</td>
<td>0.217</td>
</tr>
</tbody>
</table>
Figure 11: 3-D Multi-bay, 4 Storey Building plan

Figure 12. Substitute frame, Shear wall and frame properties and loading for 3-D, 4 storey Building

\[ l_w = 348 \times 10^{-3} b^4 \]
\[ l_c = 272 \times 10^{-6} b^4 \]
\[ a_{tw} = 740 \times 10^{-3} b^2 \]

Figure 13. B.M.D. of the equivalent shear walls For 3-D, 4 storey Building (M x Ph/10)

Figure 14. B.M.D. of the equivalent column of the substitute frame, 3-D, 4 storey Building (M x Ph/10)
The compatibility of the displacements is ensured as the lateral translations of the two different vertical members are equal at all levels as seen from the results obtained from MTProg.

From the different comparisons, it is clearly seen that, the MTProg values are very close to the reference values.

**Multi-bay Twenty Storey Structure:**

The analysis of a multi-bay twenty storey building with the same dimensions as in 3.2 was carried out using MTProg. The external applied forces are \( P/2 \) on the top floor and \( P \) on each of the others. All stories have the same height \( h \).

Comparison of the MTProg results and reference \([1]\) results are shown in Table 5 and Figures 15 and 16.

<table>
<thead>
<tr>
<th>Floor #</th>
<th>Ghali &amp; Neville</th>
<th>MTProg</th>
<th>%Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3.32 P</td>
<td>-3.37 P</td>
<td>1.506</td>
</tr>
<tr>
<td>2</td>
<td>-1.80 P</td>
<td>-1.84 P</td>
<td>2.222</td>
</tr>
<tr>
<td>3</td>
<td>-0.88 P</td>
<td>-0.92 P</td>
<td>4.545</td>
</tr>
<tr>
<td>4</td>
<td>0.08 P</td>
<td>0.05 P</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>1.04 P</td>
<td>1.01 P</td>
<td>-2.885</td>
</tr>
<tr>
<td>6</td>
<td>2.00 P</td>
<td>1.98 P</td>
<td>-1.000</td>
</tr>
<tr>
<td>7</td>
<td>2.97 P</td>
<td>2.96 P</td>
<td>-0.337</td>
</tr>
<tr>
<td>8</td>
<td>3.96 P</td>
<td>3.95 P</td>
<td>-0.253</td>
</tr>
<tr>
<td>9</td>
<td>4.97 P</td>
<td>4.97 P</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>6.01 P</td>
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<td>0</td>
</tr>
<tr>
<td>11</td>
<td>7.07 P</td>
<td>7.08 P</td>
<td>0.141</td>
</tr>
<tr>
<td>12</td>
<td>8.17 P</td>
<td>8.18 P</td>
<td>0.122</td>
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<td>13</td>
<td>9.30 P</td>
<td>9.33 P</td>
<td>0.323</td>
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<td>14</td>
<td>10.49 P</td>
<td>10.52 P</td>
<td>0.286</td>
</tr>
<tr>
<td>15</td>
<td>11.73 P</td>
<td>11.77 P</td>
<td>0.341</td>
</tr>
<tr>
<td>16</td>
<td>13.03 P</td>
<td>13.08 P</td>
<td>0.384</td>
</tr>
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<td>17</td>
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<td>14.45 P</td>
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<td>15.84 P</td>
<td>15.89 P</td>
<td>0.316</td>
</tr>
<tr>
<td>19</td>
<td>17.36 P</td>
<td>17.43 P</td>
<td>0.403</td>
</tr>
<tr>
<td>20</td>
<td>18.93 P</td>
<td>19.00 P</td>
<td>0.370</td>
</tr>
</tbody>
</table>

In floor # 4 the shear force is of a very small value (0.05 P), hence the difference was neglected. The values for all other floors are close (with a max percentage difference of 4.545).

Three Stories building with three shear walls:

The horizontal force resisted by each of the shear walls 1, 2, and 3 in a three-storey building whose plan is shown in Figure 17 is obtained as follows.
All the walls are fixed to the base. Shear deformation is considered, warping effect is ignored, Young modulus is taken as, \( E = 2.3 \, \text{G} \). The properties of the shear walls are shown in Table 6.

<table>
<thead>
<tr>
<th>Wall</th>
<th>( I_u )</th>
<th>( I_v )</th>
<th>( J )</th>
<th>( a_{ru} )</th>
<th>( a_{rv} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall 1</td>
<td>0.03413 b^4</td>
<td>0.03413 b^4</td>
<td>0.0512 b^4</td>
<td>0.16 b^2</td>
<td>0.16 b^2</td>
</tr>
<tr>
<td>Wall 2 or 3</td>
<td>0.0342 b^4</td>
<td>0.00013 b^4</td>
<td>0.00053 b^4</td>
<td>0.133 b^2</td>
<td>0.133 b^2</td>
</tr>
</tbody>
</table>

The global displacements of the shaft center as presented in ref.[1] are:

\[ \{ D \} = \{ x_1 , x_2 , x_3 , y_1 , y_2 , y_3 , rz_1 , rz_2 , rz_3 \} = P/Eh \{ -0.154 , -0.105 , -0.060 , -354.258 , -210.885 , -79.686 , 39.09/h , 23.31/h , 8.82/h \} \], and the forces resisted by the various shear walls are shown in Figure 18.

Comparison of the results of the MTProg and reference [1] are shown in Tables 7 through 10, which show almost identical results.
Figure 18. Forces resisted by the various shear walls, ref\textsuperscript{[1]}

Figure 19. 12m x 12m floor plan for 15 storey, square building.
Fifteen storey square building subjected to unsymmetrical lateral loading:
The plan shown in Figure 19 is for a 12m x 12m floor slab of thickness = 0.25 m. The building is composed of 15 floors of floor height = 3.5 m for all floors except the lower floor which is of height = 5.5 m. All building members are concrete of modulus of elasticity, \( E = 21718500 \) kN/m\(^2\) and Poisson's ratio, \( v = 0.17 \).

The section properties of the vertical elements (in meters) are:
Columns: Corners: 0.60 x 0.60 and Interior: 0.85 x 0.85
Shear walls: The lower 7 floors: 0.30 x 3.00 and The upper 8 floors: 0.25 x 3.00

The building is subjected to the lateral loads shown in Figure 19, (30 kN and 50 kN) at all floor levels.

The building has been analyzed by using MTPro and the accuracy of the results is verified by using the structural analysis packages STAADPro 2004\(^{[14]}\) and ETABS\(^{[15]}\).

The floor out of plane stiffness is calculated by constructing the global stiffness matrix of one floor slab (plates and stiff members representing the rigid parts corresponding to the shear walls). The matrix is rearranged and partitioned into two parts, one corresponding to the supported DOFs and the other corresponding to the free DOFs.

The matrix is then condensed into a small matrix that represents the rotation stiffness of the supported DOFs with all the remaining DOFs translating and rotating freely. This is done only once for floors with identical structural members. The proposed method is very fast compared with the finite element packages.

The lateral displacements from all analyses are compared for the various walls, the agreement is found good.

Table 8. Comparison of the Global Displacements of the middle floor (D x P/Eh ).

<table>
<thead>
<tr>
<th>Package</th>
<th>Global X</th>
<th>Global Y</th>
<th>Global Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ghali &amp; Neville</td>
<td>-0.105</td>
<td>-210.885</td>
<td>23.31 / h</td>
</tr>
<tr>
<td>MTProg</td>
<td>-0.105</td>
<td>-210.887</td>
<td>23.317 / h</td>
</tr>
<tr>
<td>%Difference</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 9. Comparison of the Global Displacements of the bottom floor (D x P/Eh ).

<table>
<thead>
<tr>
<th>Package</th>
<th>Global X</th>
<th>Global Y</th>
<th>Global Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ghali &amp; Neville</td>
<td>-0.060</td>
<td>-79.686</td>
<td>8.82 / h</td>
</tr>
<tr>
<td>MTProg</td>
<td>-0.060</td>
<td>-79.687</td>
<td>8.829 / h</td>
</tr>
<tr>
<td>%Difference</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Comparison of bending moments in kN.m and shear forces in kN, for the different programs are shown in Table 11 and Figures 23 through 30.

In all cases the comparison, shows the difference to be very small for large stress values (shear forces and bending moments in shear walls 1,2 & 3). The largest percentage difference is found in shear wall number 4, but this resists very small stresses compared with its section.

Figure 20. Displacements of the origin in x-direction.
Table 10. Comparison of the Shear Walls Forces and torsion in the different shear walls at the bottom floor.

<table>
<thead>
<tr>
<th></th>
<th>Shear wall 1</th>
<th>Shear wall 2</th>
<th>Shear wall 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forces</td>
<td>Fu</td>
<td>Fv</td>
<td>Tors.</td>
</tr>
<tr>
<td>Ghali &amp; Neville</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fu</td>
<td>0</td>
<td>-2.089P</td>
<td>0.1966Ph</td>
</tr>
<tr>
<td>Fv</td>
<td>-2.089P</td>
<td>0.003Ph</td>
<td>0.043P</td>
</tr>
<tr>
<td>Tors.</td>
<td>0.1966Ph</td>
<td>0.003Ph</td>
<td>0.043P</td>
</tr>
<tr>
<td>%Difference</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 21. Displacements of the origin in y-direction.

Figure 22. Rotations in radians of the origin.

Table 11. Comparison of the maximum shear force (kN) and bending moment (kN.m):

<table>
<thead>
<tr>
<th>Wall #</th>
<th>Shear Wall 1</th>
<th>Shear Wall 2</th>
<th>Shear Wall 3</th>
<th>Shear Wall 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shear</td>
<td>Moment</td>
<td>Shear</td>
<td>Moment</td>
</tr>
<tr>
<td>Package</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MTProg</td>
<td>155.354</td>
<td>1583.725</td>
<td>500.26</td>
<td>5546.552</td>
</tr>
<tr>
<td>Etabs thick. Slab</td>
<td>156.66</td>
<td>1604.84</td>
<td>494.60</td>
<td>5424.34</td>
</tr>
<tr>
<td>Etabs thin slab</td>
<td>157.74</td>
<td>1697.90</td>
<td>497.21</td>
<td>5727.17</td>
</tr>
</tbody>
</table>
Figure 23. Comparison of B.M.D. for shear wall #1

Figure 24. Comparison of S.F.D. for shear wall #1

Figure 25. Comparison of B.M.D. for shear wall #2

Figure 26. Comparison of S.F.D. for shear wall #2
Comparison of the Computer Storage and the Running Time:
In order to show the visibility of the proposed transformation method in terms of computer storage and computational running time, the floor slab was idealized by 24 x 24 finite elements with 9 vertical members (shear walls and columns), Figure 19, for three models of 30, 40 and 50 floors are studied using the simplified and conventional matrix methods.

Computer storage comparison:
As an example the total real numbers required for the storage of the global stiffness matrix for a building with the same floor and of total N floors is equal to:

a) For conventional matrix methods of analysis:
   \[ S_1 = (25 \times 25 \times 6 \times N)^2 \]

b) For Moment Transformation Method:
   \[ S_2 = (25 \times 25 \times 3)^2 \]
   … Storage for one floor with 3 DOFs per joint.
+ \[ (9x2)^2 \times N \] ... Rotation stiffness of vertical members in the 2 principal directions.
+ \[ (9x2)^2 \times N \] ... Carry-over moment matrix in the 2 principal directions.
+ \[ (9x2)^2 \times (N+1) \] ... Floor Level rotation stiffness.

For double precision, one real number requires 8 bytes storage. Figure 31 shows comparison curves for computer storage.

Figure 31. Comparison curves of the computer storage for the transformation and the conventional methods of analysis.

**Running time comparison:**
The computational time required for the analyses of the three models of 30, 40 and 50 floors, by using the two programs MTProg and STAADPro package, are shown in Table 12 and Figure 32.

Table 12. Computing time required for the analysis of the models for the programs MTProg and StaadPro.

<table>
<thead>
<tr>
<th>Total Floors</th>
<th>1-STAADPro (sec.)</th>
<th>2-MTProg. (sec.)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>854</td>
<td>19.17</td>
<td>44.5</td>
</tr>
<tr>
<td>40</td>
<td>1133</td>
<td>22.72</td>
<td>49.8</td>
</tr>
<tr>
<td>50</td>
<td>1165</td>
<td>42.65</td>
<td>27.3</td>
</tr>
</tbody>
</table>

Figure 32. Comparison of the computing time for the programs MTProg and StaadPro.

From the above comparisons, it is clearly seen that the ‘exact’ methods of analysis of tall buildings by using personal computers is not advisable in a preliminary design stage when the structure has to be modified rapidly and frequently. This can be performed easily and with a low cost by using the proposed simplified analysis method.

These comparisons have been carried by using computer: Dell Inspiron 6400, genuine Intel(R) CPU Dual Core, T2080 @ 1.73 GHz, 795 MHz 0.99 GB of RAM.

**Conclusion**
(1) The Moment Transformation Method is suitable for the analysis of tall buildings with shear walls, wherein the axial deformations are negligible.
(2) The Method yields adequate results and the accuracy as compared to exact analysis is very good.
(3) The saving in computer storage and computing time provided by the proposed program MTProg allows rapid re-analysis of the building to be accomplished in the preliminary analysis and design stages.
(4) The simplicity in programming the method, when compared with the difficulty in obtaining reliable packages, is an added advantage.
(5) The ease in data preparation and interpretation of final results, compared with finite element packages, is one of the main advantages of the method.

For future research it is recommended that:
(1) Dynamic characteristics of the buildings could be included.
(2) In order to establish a more accurate analysis for tall buildings the axial deformation of the vertical members should be incorporated.
(3) In the existence of the axial deformation of the vertical members, the following research areas can be looked into:
   a. Capability of analyzing taller buildings, by development of modules to analyze tube and outrigger buildings systems.
   b. Non-linear analysis (P–Delta effects) and elastic stability of buildings.
c. Columns shortening in tall buildings for both steel and concrete structures.

References:


