Steady State Stability of the Multi-Machine National Electric Grid System (Sudan)

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Abstract:
This paper presents a comprehensive approach to evaluate steady state stability performance for the National Grid for Electricity. A data bank for stability studies has been collected. This has been used to construct appropriate mathematical models for the system power plants and their auxiliaries. Detailed simulation and analysis for the system have been carried out. The effect of reactive power compensation on system performance has been investigated. System mode identification via an eigenvalue approach has been accomplished.

1. Introduction:

Power system stability has been an area of study from the early days of power generation and transmission [1]. This has become an area of concern as power systems over large geographic areas have been interconnected. As the demand for electrical power has been increased, very long transmission lines with modern control systems are required [2]. Hence power systems have become more complex, and this control for maintaining stability has become very complicated. This has resulted in detrimental effects on transient and steady state stability of electric power system. With the growth of this problem, sophisticated control equipment and protection schemes have been added to the power system to improve stability. As a result the analysis of the power system has become more complicated, but better mathematical models and their use in computer simulation have also brought increased understanding.

2. MATHEMATICAL MODELS:

The mathematical models, which are used to represent the synchronous machine and associated control systems in conventional steady state programs, involve certain approximations. These approximations arise partly out of a wish to determine the electromechanical dynamic performance of very large, multi-machine power systems. Accordingly, it is desirable to reduce the complexity of the generator models and system representation to the point. Where a large number of machines can be represented but within the constraint, that an acceptable level of accuracy is obtained which will lead to meaningful judgments about the design and operation of the system [3].

First of all, the transmission network, which couples the machines together, is represented by its steady state positive sequence equivalent comprised of lumped parameters at the nominal steady state frequency of the system. This is generally acceptable because the electromechanical oscillations of the machines are of much lower frequency than the fundamental frequency of the system. Then, the network can be described by algebraic equations, and the solution of the network equations is greatly simplified. A steady state load flow solution of the network equations is performed taking into account the instantaneous node voltage as provided by a solution of the swing equation at each dynamics node, and possibly accounting for nonlinear loads as appropriate.

The synchronous machine models are based on the Park's equations formulation whereby the differential equations relating instantaneous currents, voltages and flux linkages in the stator (R-Y-B) are transformed into a rotor reference frame (d-q-o) resulting in differential equations with constant rather than time vary coefficients [3,4]. The solutions of these equations, together with others

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representing field and damper circuits, yield instantaneous flux and current quantities, which are used to determine the electromagnetic torque developed by the machine. This torque is then used in the basic swing equation, in conjunction with mechanical and damping torque's acting on the rotor, to determine the rotor motion of each machine being represented.

The influence of the excitation control system is incorporated in the differential equations associated with the field circuit of the machine. Standard models represent the response of the excitation system to feedback voltage regulator control. The prime mover and speed governor control is also of standard models [3, 4].

The differential and algebraic equations describing the performance of a power system are basically nonlinear and can be described by a set of first order differential equations [2]:

$$\left.\begin{aligned} pX &= f(X)+g(U) \\ Y &= h(X)+K(U) \end{aligned}\right\} \qquad (1)$$

Where X, U and Y are the state, input and algebraic vectors of suitable dimensions and f, g, h and K are vector functions of system parameters and operating conditions.

In steady state stability studies, since it is difficult to write the equations directly in the form:

$$\left.\begin{aligned} pX &= [A][X]+[B][U] \\ Y &= [C][X]+[D][U] \end{aligned}\right\} \qquad (2)$$

The equation can be written in the following form:

$$[P]\begin{bmatrix} pX \\ Y \end{bmatrix} = [Q][X]+[R][U]$$

Where P, Q and R are real constant matrices with appropriate dimensions. The entries of these matrices are function of all the system parameters and depend on the operating conditions.

2.1 Network Equations:

A load flow solution of the power system in Figure (1) is to be obtained first and the steady state values of the system variables are then calculated. If the system has a non-synchronous load, which can be represented by a constant admittance, the latter is added to the self-admittance of the node at which the load is connected. All the node loads are then eliminated using the will known Kron reduction method. Thus node equations of the reduced network, which contains only machine nodes, may be written as: $[I]=[Y][V]$ (3)

2.2 System Transformation:

The equations of each machine are usually referred to its rotor d and q axes. In steady state the axes of the machines rotate at a constant speed with angular differences as shown in Figure (2) [5].

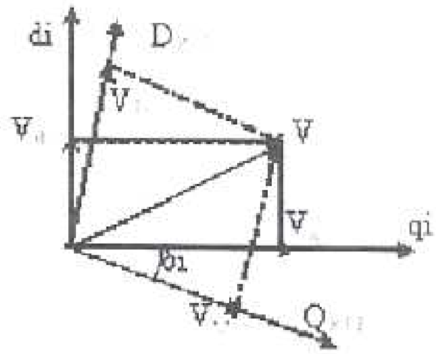

Figure (2) Frames of reference for phasor V_i

From Figure (2) by inspection we can show that

$$V_{Qi} + jV_{Di} = (V_{qi}\cos\delta_i - V_{di}\sin\delta_i) + j(V_{qi}\sin\delta_i + V_{di}\cos\delta_i)$$

Therefore the equations referred to the common reference D and Q frame are [4]:

$$\begin{bmatrix} V_{Di} \\ V_{Qi} \end{bmatrix} = \begin{bmatrix} \cos\delta_i & \sin\delta_i \\ -\sin\delta_i & \cos\delta_i \end{bmatrix} \begin{bmatrix} V_{di} \\ V_{qi} \end{bmatrix}$$

In symbolic form the above equation can be written as $V_N = T\, V_m$ (4)

Equation (3) can therefore be expressed in the double real form

$$\begin{bmatrix} I_{D1} \\ I_{Q1} \\ \cdot \\ \cdot \\ I_{Di} \\ I_{Qi} \end{bmatrix} = \begin{bmatrix} G_{11} & -B_{11} & \cdot & \cdot & G_{1i} & -B_{1i} \\ B_{11} & G_{11} & \cdot & \cdot & B_{1i} & G_{1i} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ G_{i1} & -B_{i1} & \cdot & \cdot & G_{ii} & -B_{ii} \\ B_{i1} & G_{i1} & \cdot & \cdot & B_{ii} & G_{ii} \end{bmatrix} \begin{bmatrix} V_{D1} \\ V_{Q1} \\ \cdot \\ \cdot \\ V_{Di} \\ V_{Qi} \end{bmatrix}$$

Or in the symbolic form:

$$I_N = Y_N\, V_N \qquad (5)$$

The Common reference frame may be chosen arbitrary. The system variables are referred to the common reference frame of the network, we follow Undrill's [6] assumption considering the network frequency is always identical to that of one arbitrary selected machine.

3. Linearized Model of Multi-machine System:

The method of obtaining a Linearized model for multi-machine system is explained here for a two-machine system in order to clarify the notations. Extension to the systems with more machines can easily be made [7].

3.1 Linearized Machine Equations:

$$p\Delta\delta = \Delta\omega \quad \ldots\ldots(6)$$

$$p\Delta\omega - \frac{\omega_o}{2H}\psi_q \Delta i_d + \frac{\omega_o}{2H}\psi_d \Delta i_q =$$

$$-\frac{\omega_o}{2H} I_q \Delta\psi_d + \frac{\omega_o}{2H} I_d \Delta\psi_q + \frac{\omega_o}{2H}\Delta T_m \quad \ldots\ldots 7$$

Equation (7) can be augmented for i^{th} generators and hence leads to:

$$U\, p\Delta\omega + \theta\, \Delta i_{dq} = \gamma\, \Delta\psi + K\, U_g \quad \ldots\ldots(8)$$

Where θ, γ and K are sub-matrices with appropriate dimensions illustrated in Appendix (A).

$$\left.\begin{aligned}
&\frac{p\Delta\psi_F}{\omega_o} + R_F \Delta i_F = \frac{R_F}{M_F}\Delta E_{FD} \\
&\frac{p\Delta\psi_d}{\omega_o} - \Delta V_d - R_a \Delta i_d = \frac{\psi_q}{\omega_o}\Delta\omega + \Delta\psi_q \\
&\frac{p\Delta\psi_D}{\omega_o} + R_D \Delta i_D = 0 \\
&\frac{p\Delta\psi_q}{\omega_o} - \Delta V_q - R_a \Delta i_q = -\frac{\psi_d}{\omega_o}\Delta\omega - \Delta\psi_d \\
&\frac{p\Delta\psi_Q}{\omega_o} + R_Q \Delta i_Q = 0
\end{aligned}\right\} \quad \ldots\ldots(9)$$

The flux equation for the i^{th} generators can be obtained in a linear form as follows:

$$\xi\, p\Delta\psi + \lambda\, \Delta V_{dq} + \phi\, \Delta i = \gamma\gamma\, \Delta\omega + \varepsilon\, \Delta\psi_{dq} + \Delta U_e \quad \ldots\ldots(10)$$

Where ξ, λ, Φ, γγ and ε are sub-matrices with appropriate dimension illustrated in Appendix (A).

The current vectors in both machine types are related to the flux vector by

$$\Delta i = L^{-1}\, \Delta\psi \quad \ldots\ldots(11)$$

$$L = \begin{bmatrix} L_F & -M_F & M_{FD} & 0 & 0 \\ M_F & -L_d & M_D & 0 & 0 \\ M_{FD} & -M_D & L_D & 0 & 0 \\ 0 & 0 & 0 & -L_q & M_Q \\ & & & -M_Q & L_Q \end{bmatrix}$$

Moreover, the terminal voltages are included for the purpose of up-grading the control systems as $V_t^2 = V_d^2 + V_q^2$ the linear form of this equation is obtained for i^{th} generators as [5,7]:

$$\Delta V_t - \frac{V_d}{V_t}\Delta V_d - \frac{V_q}{V_t}\Delta V_q = 0$$

Or as follows:

$$\Delta V_t + KK\, \Delta V_{dq} = 0 \quad \ldots\ldots(12)$$

For small perturbation to equation (4) we have:

$$\begin{bmatrix} \Delta V_{Di} \\ \Delta V_{Qi} \end{bmatrix} = \begin{bmatrix} \cos\delta_i & \sin\delta_i \\ -\sin\delta_i & \cos\delta_i \end{bmatrix} \begin{bmatrix} \Delta V_{di} \\ \Delta V_{qi} \end{bmatrix} + \begin{bmatrix} V_{qi}\cos\delta_i - V_{di}\sin\delta_i \\ -V_{qi}\sin\delta_i - V_{di}\cos\delta_i \end{bmatrix} [\Delta\delta_i] \quad \ldots\ldots 13$$

Which may be rewritten in symbolic form as:

$$\Delta V_N = T\, \Delta V_m + D\, \Delta\delta \quad \ldots\ldots(14)$$

The incremental form of the network node equation (5) is

$$\Delta I_N = Y_N\, \Delta V_N \quad \ldots\ldots(15)$$

From the power invariance theorem Kron

$$i_m = T^t\, I_N \quad \ldots\ldots(16)$$

Where i_m is the vector of the machine currents and T^t is the transpose of the transformation matrix. For small perturbation, we have

$$\begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta i_{d2} \\ \Delta i_{q2} \end{bmatrix} = [T]^t \begin{bmatrix} \Delta i_{D1} \\ \Delta i_{Q1} \\ \Delta i_{D2} \\ \Delta i_{Q2} \end{bmatrix} + \begin{bmatrix} i_{q1} & 0 \\ -i_{d1} & 0 \\ 0 & i_{q2} \\ 0 & -i_{d2} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \end{bmatrix}$$

Or in compact form

$$\Delta i_m = T^t\, \Delta I_N + E\, \Delta\delta \quad \ldots\ldots(17)$$

Eliminating ΔI_N and ΔV_N form equation (14), (15) and (17) and solved for ΔV_m, we have:

$$\Delta V_m = G\, \Delta i_m + Q\, \Delta\delta \quad \ldots\ldots(18)$$

Where $G = T^t Y^{-1}_N T$ and $Q = -G\,E - T^t D$

3.2 Open Loop Overall Model:

The overall linear model is obtained by combining the machines and network equation (8), (10), (11), (12) and (18) as follows:

$$\begin{bmatrix} U & 0 & 0 & 0 & 0 & 0 \\ 0 & U & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi & \lambda & \phi & 0 \\ 0 & 0 & 0 & U & G & 0 \\ 0 & 0 & 0 & 0 & U & 0 \\ 0 & 0 & 0 & KK & 0 & U \end{bmatrix} \begin{bmatrix} p\Delta\delta \\ p\Delta\omega \\ p\Delta\psi \\ \Delta V_{dq} \\ \Delta I \\ \Delta V_t \end{bmatrix} = \begin{bmatrix} 0 & U & 0 \\ 0 & 0 & \gamma \\ 0 & \gamma\gamma & \varepsilon \\ Q & 0 & 0 \\ 0 & 0 & L^{-1} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta\psi \end{bmatrix}$$

T

$$+ \begin{bmatrix} 0 & 0 \\ 0 & K \\ U & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_e \\ U_g \end{bmatrix} \quad \ldots\ldots 19$$

he above equation can be written simply as:

$$[P]\begin{bmatrix} p\Delta X \\ \Delta Y \end{bmatrix} = [Q][\Delta X] + [R][U] \quad \ldots\ldots 20$$

Where $[\Delta X]^t = [\Delta\delta\ \Delta\omega\ \Delta\psi]$ and $[\Delta Y]^t = [\Delta V_{dq}\ \Delta I\ \Delta V_t]$

The order of the P matrix is 15m where m is the number machines. Using matrix manipulation, the linear time-invariant state space model (20) can be obtained in the following form $p\Delta X = A \Delta X + B U$. Where

A is 7m×7m-system matrix at open loop system

A is 12m×12m-system matrix at close loop system

B is 7m×2m input matrix at open loop system

B is 12m×(n×m) input matrix at close loop system

Where n is the number of input.

4. Control System:

The excitation system for all generators is a static excitation system [8]. The Turbine – Governor models are concerned with the representation of hydraulic, steam and gas turbines and their speed governing systems for power system stability studies. The intent is to indicate the origin of the models currently in wide use, to bring about a greater understanding of when these models may and may not be used and promote standardization of nomenclature [9]. Three basic models are:

1) Turbine and speed governing system for hydro turbine.

2) Turbine and speed governing system for steam turbine.

3) Turbine and speed governing system for gas turbine.

5. Digital Computer Implementation:

The programming of the above procedure for constructing [A] is straightforward and has been implemented in FORTRAN. The present capacity of the program is nine machines, but this may be expanded in computers with large core memories. The program forms the complete matrices before allowing the user to be specifying the reference machine and to specify which variables, if any, are to be deleted. This allows any combination of generators, motors, or synchronous condensers to be studies since the only distinction between these machines is in the prime mover system and steady state solution. Non-synchronous loads must be represented as shunt admittances in the original Transmission Network representation [6].

After forming [A] a standard subroutine is used to compute the complex eigenvalues of [A]. The eigenvalues of a linear dynamical system correspond to its natural modes of response, with each real part giving the reciprocal decay time constant or damping coefficient of a mode and each pair of imaginary parts giving a natural angular frequency [6].

6. Multi-machine System Analysis:

The total number of eigenvalues describing the system performance is 105. It is very important to identify these eigenvalues to obtain a measure of the system stability by which unstable modes can be stabilized. The most important modes are those related to the torque-angle loop. The interactions between the field circuits and the excitation systems are also of great importance since they may produce unstable modes at some cases. The upper eighteen eigenvalues of Table (1) column (3) are largely stable and related to the system fast transients. The excitation system eigenvalues given in Table (1) column (3) can be identify by varying the excitation system parameters. The last sets of eigenvalues in Table (1) column (3) are related to the turbine-governor system. The first eighteen eigenvalues given in Table (1) column (3) related to the network transients in the R, Y, B axis which become approximately 50 Hz in the d-q variables infinite. Since the network has been modeled by algebraic equations, so network transients have not been represented in multi-machine simulation. This results in that these frequencies did not appear in the multi-machine case. Usually network transients are important in studying sub-synchronous resonance and shaft dynamics. These transient modes are well damped. These are followed by eigenvalues ($\lambda_{1,2}$) associated with the natural mechanical oscillations of the rotor i.e. the torque angle loop eigenvalues. These eigenvalues pairs are different from those corresponding to each machine connected to infinite bus. This is because of the interaction between the whole set of eigenvalues and also due the effect of system network. The damping of these eigenvalues in the case of single machine is more than that of multi-machine case due to strong transmission line estimated in the first case. These are followed by the set of eigenvalues related to the interaction between the field circuit and the excitation systems. Some interactions also occurred between the other system eigenvalues. The last two sets of eigenvalues shown in section four and five are related to the excitation and turbine-governor systems. These eigenvalues are similar to the case of single machine with very little difference.

In the case of machine only the system is represented by nine power plants neglecting the effect of both the excitation and turbine-governor systems. The number of eigenvalues in this case is 63 and they are given in Table (1) column (2). Comparing the eigenvalues of this case with close loop system, it is found that the damping of the dominant eigenvalue has been increased. Also the damping of three torque angle loops have been increased and one decreased while the other are unchanged. This shows also the importance of the accuracy in system simulation.

6.1 The Effect of Reactive Power Compensation on Steady State Stability:

Reactive power compensation can contribute to the enhancement of the power system dynamic performance. Normally, voltage regulation is the primary mode of control, and this improves voltage stability and transient stability. However, the contribution of reactive power compensation to the damping of system oscillations resulting from voltage regulation alone is usually small, supplementary control is necessary to achieve significant damping. The effectiveness of reactive power compensation in enhancing steady state stability depends on the location of the static compensators. Table (1) column (4) illustrated the effect of reactive power compensation on $\lambda_{1,2}$ which, increased the damping and decreased the oscillation. The effect on $\lambda_{21,\ 22}$ and $\lambda_{23,\ 24}$ is to increase the damping only.

6.2 The Effect of Controller Parameters:

In this section the effects of controller parameters on system performance are studied. The problem of selecting the appropriate values of controller parameters for the multi-machine power system is considered. The aim is to study how to improve the steady state stability of the interconnected power system by proper choice of system control parameters.

For the control system to be effective in improving power system stability their parameters must be properly chosen and these parameters must be optimally coordinated to give the best overall dynamic performance [4].

It is important to report here that changing the parameters of any control system will not affect the modes of that machine only but will affect also the other machine modes. This is due to the inter-connection between the different modes of different groups in the system. These interactions between the modes are very dangerous since many unstable oscillations can be generated.

The system eigenvalues with the effect of controller are given in Table (1). Column (5) illustrated the system eigenvalues when the parameters of exciter and turbine-governor changing to the most suitable values.

6.2.1 The Effect of Excitation System Parameters:

The effect of changing K_A of PP1 on system performance is that increasing K_A will increase the damping in the eigenvalue λ_{70}, which is related to the excitation. Also increasing K_A will reduce the damping in the eigenvalue λ_{68}. Also the effect of amplifier time constant T_A of PP2 is shown in Figure (3). This figure shows that changing this time constant will affect several eigenvalues, which are $\lambda_{27,28}$, $\lambda_{31,32}$, $\lambda_{33,34}$ and $\lambda_{35,36}$ increasing T_A will reduce the damping of eigenvalues. The most appropriate range for T_A is from 0.1 up to 1.0. Decreasing T_A less than 0.1 will decrease the damping of the eigenvalues and also increase the oscillation.

The effect of K_F on system performance is that increasing K_F will increase the damping of eigenvalues $\lambda_{27,28}$ and $\lambda_{35,36}$. Also reducing K_F will reduce the same eigenvalue.

The effect of time constant of the excitation feedback stabilizer T_F is that increasing T_F, will decrease the damping in the following eigenvalues λ_{72}, λ_{77}.

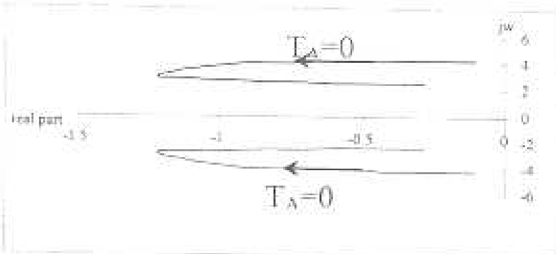

Figure (3) Effect of T_A on λ_{35}, λ_{36} mode

6.2.2 The Effect of Turbine-Governor Parameters:

The effect of the time constant T_1 of hydro turbine- governor system is shown in Figure (4). This figure shows that increasing T_1 will increase the damping and reduced the oscillation of $\lambda_{27,28}$. Also increasing T_1 will decrease the damping and the oscillation of $\lambda_{35,36}$, which are related to torque angle loop.

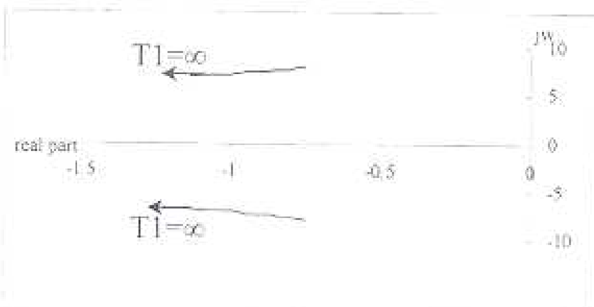

Figure (5) Effect of K_T of Gas on System modes

The effect of changing the combustion chamber time constant T_2 of gas turbine is shown in Figure (6). This figure shows that increasing T_2 will affect the following set of eigenvalues: λ_{88}, λ_{89}, λ_{90}, λ_{91}, λ_{92}, λ_{93}, λ_{94} and λ_{95}. The damping in this eigenvalues are improved as T_2 decreased. Also increasing T_2 will increase the oscillation of this eigenvalues, which related to turbine-governor system.

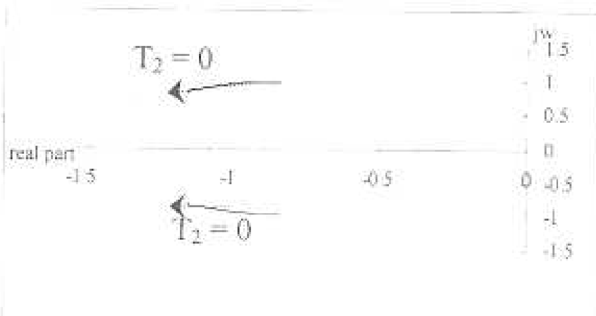

Figure (6) Effect of T_2 of Gas on System Modes

7. Conclusions:

This paper presents an overall steady state stability study for the National Grid, and demonstrates the importance of steady state stability analysis for recently developing power systems. The importance of data availability and accuracy regarding system configuration as well as system physical apparatuses for constructing appropriate computer models has been highlighted. It has been shown that system parameters as well as operating conditions affect system performance. Also, tuning these parameters is an effective tool to improve the system overall stability performance.

8. References:

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9. Appendix (A):

$$\xi = \begin{bmatrix} \xi_1 & . & 0 \\ . & . & . \\ 0 & . & \xi_n \end{bmatrix}, \xi_n = \begin{bmatrix} \frac{1}{\omega_o} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\omega_o} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\omega_o} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\omega_o} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\omega_o} \end{bmatrix}$$

$$\theta = \begin{bmatrix} \theta_1 & . & 0 \\ . & . & . \\ 0 & . & \theta_n \end{bmatrix}, \theta_n = \begin{bmatrix} 0 & \frac{-\omega_o}{2H_n}\psi_{qn} & 0 & \frac{-\omega_o}{2H_n}\psi_{dn} & 0 \end{bmatrix}$$

$$\gamma = \begin{bmatrix} \gamma_1 & . & 0 \\ . & . & . \\ 0 & . & \gamma_n \end{bmatrix}, \gamma_n = \begin{bmatrix} 0 & \frac{-\omega_o}{2H_n}I_{qn} & 0 & \frac{-\omega_o}{2H_n}I_{dn} & 0 \end{bmatrix}$$

$$\phi = \begin{bmatrix} R_1 & . & 0 \\ . & . & . \\ 0 & . & R_n \end{bmatrix}, R_n = \begin{bmatrix} R_{Fn} & 0 & 0 & 0 & 0 \\ 0 & -R_{An} & 0 & 0 & 0 \\ 0 & 0 & R_{Dn} & 0 & 0 \\ 0 & 0 & 0 & -R_{An} & 0 \\ 0 & 0 & 0 & 0 & R_{Qn} \end{bmatrix}$$

$$\lambda = \begin{bmatrix} \lambda_1 & . & 0 \\ . & . & . \\ 0 & . & \lambda_n \end{bmatrix}, \lambda_n = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}, \gamma\gamma = \begin{bmatrix} \gamma\gamma_1 & . & 0 \\ . & . & . \\ 0 & . & \gamma\gamma_n \end{bmatrix}$$

$$\gamma\gamma_n = \begin{bmatrix} 0 \\ \frac{\psi_{qn}}{\omega_o} \\ 0 \\ \frac{-\psi_{dn}}{\omega_o} \\ 0 \end{bmatrix}, \varepsilon = \begin{bmatrix} \varepsilon_1 & . & 0 \\ . & . & . \\ 0 & . & \varepsilon_n \end{bmatrix}, \varepsilon_n = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$KK = \begin{bmatrix} KK_1 & . & 0 \\ . & . & . \\ 0 & . & KK_n \end{bmatrix}, KK_n = \begin{bmatrix} \frac{-V_{dn}}{V_{tn}} & \frac{-V_{qn}}{V_{tn}} \end{bmatrix}$$

$$K = \begin{bmatrix} K_1 & . & 0 \\ . & . & . \\ 0 & . & K_n \end{bmatrix}, K_n = \begin{bmatrix} \frac{\omega_o}{2H_1} & 0 \\ 0 & \frac{\omega_o}{2H_n} \end{bmatrix}$$