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The Solution of Collinearity Condition Equations With 6-Terms via 10-Terms

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Abstract: Collinearity condition equations are one of the most fundamental equations in analytical and digital photogrammetry. They have different forms and applications in photogrammetry, such as the computation of internal and external orientation parameters of the photograph and computation of ground coordinates of points that can be identified the successive overlapping photos. This study discusses the results of applying two terms of collinearity condition equations, one consists of 6-terms and the other consists of 10-terms on a photo a stereo-pair, consists of 20 control points, acquired by an aerial camera. It was found to be that for the metric camera the two algorithms relatively produces the same precision. Therefore, in aerial photogrammetry, it is sufficiently enough to use the model of 6-terms collinearity condition equations for topographic mapping. This will certainly reduce the cost of establishing more ground control points.

Key words: Photogrammetry, topographic mapping, collinearity

INTRODUCTION

Collinearity condition equations are a projective coordinate transformation algorithm, which relates the three dimensional ground coordinate system (X, Y and Z) to the two dimensional (x and y) photo coordinate system through the three dimensional rotational matrix as follows (El-Beik and Masaad, 1993):

$$\left.\begin{aligned} x &= -f\left[\frac{(m_{11}(X_A - X_L)m_{12}(Y_A - Y_L)m_{13}(Z_A - Z_L))}{(m_{31}(X_A - X_L)m_{32}(Y_A - Y_L)m_{33}(Z_A - Z_L))}\right] \\ y &= -f\left[\frac{(m_{21}(X_A - X_L)m_{22}(Y_A - Y_L)m_{23}(Z_A - Z_L))}{(m_{31}(X_A - X_L)m_{32}(Y_A - Y_L)m_{33}(Z_A - Z_L))}\right] \end{aligned}\right\} \tag{1}$$

Where,

- x, y : The photo coordinates of the control points.
- X_A, Y_A, Z_A : The object space coordinates of the control points.
- X_L, Y_L, Z_L : The object space coordinates of the perspective center.
- f : The focal length of the camera.

6-Terms algorithm: The 6-terms collinearity condition equations consists of independent quantities; these are the image coordinates (x_a, y_a, f), the object space coordinates (X_A, Y_A, Z_A), the projection center coordinates (X_L, Y_L, Z_L) and three rotation elements (ω, φ and κ) which are implicit in the quantities $m_{11},\ldots,m_{33}$ (Wolf, 1983).

The focal length f is usually a known quantity and x_a, y_a are generally observed quantities. If sufficient points of known object space coordinates are observed then the unknown tilts (ω, φ κ) and the projection center coordinates (X_L, Y_L, Z_L) may be derived. These then present the solution of the space resection problem.

Since, each ground control point, whose image appears in the stereo-pair, provides two equations, then at least three control points are required to solve for the unknowns (Burnside, 1988). Moreover, linearization of the system equations is required to generate a solution for non-linear system.

10-Terms algorithm: In computational photogrammetry, image coordinates are usually measured in an arbitrary image coordinates system (Ali, 1992). For example comparator coordinates system in analytical photogrammetry, or in pixel coordinates system in digital photogrammetry. Therefore, for an arbitrary coordinates to be processed by collinearity conditions equations it must be transformed into image coordinate system.

Traditionally the transformation from arbitrary coordinates to image coordinates system is accomplished by using the two-dimensional affine transformation algorithm as follows:

$$\left.\begin{aligned} x' &= a_1 + a_2x + a_3y \\ y' &= a_4 + a_5x + a_6y \end{aligned}\right\} \tag{2}$$

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Where:

x, y	:	The measured photo coordinates.
x', y'	:	The projected photo coordinates.
$a_1, a_2, \ldots, a_6$	:	The transformation parameters.

This transformation, account for translation, differential scale variation, non-perpendicularity of the image axes and rotations in x', y' planes.

By incorporating the affine transformation equations in Eq. 2 and the collinearity condition equations in Eq. 1 the resulted Equations (El-Biek and Masaad, 1993) are:

$$\left.\begin{aligned} a_1 + a_2x + a_3y &= -f\left[\frac{m_{11}(X_A - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_L - Z_L)}\right] \\ a_4 + a_5x + a_6y &= -f\left[\frac{m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}\right] \end{aligned}\right\} \quad (3)$$

The assumption that photo-coordinates system is parallel to the comparator coordinates system (i.e. rotational angle is equal to zero) and the systematic errors are corrected will reduce the model to be as follows:

$$\left.\begin{aligned} x' &= s_x(x - x_p) \\ y' &= s_y(y - y_p) \end{aligned}\right\} \quad (4)$$

Where,

x, y	:	The image coordinates with the principal point as origin.
s_x, s_y	:	The differential scale factors along the x-axis and the y-axis, respectively.
x_p, y_p	:	The coordinates of the principal point in the comparator coordinates system.

Substitution of Eq. 4 in 3 will produce the following equation:

$$\left.\begin{aligned} s_x(x - x_p) &= -f\left[\frac{m_{11}(X_A - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}\right] \\ s_y(y - y_p) &= -f\left[\frac{m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}\right] \end{aligned}\right\} \quad (5)$$

By dividing the first above equation by s_x and the second equation by s_y, we can get:

$$\left.\begin{aligned} (x - x_p) &= -f_x\left[\frac{m_{11}(X_A - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}\right] \\ (y - y_p) &= -f_y\left[\frac{m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}\right] \end{aligned}\right\} \quad (6)$$

Table 1: Actual ground space coordinates

	Computed space coordinates (m)		
Point	X	Y	Z
1	52802.60	45639.63	1085.89
2	53104.09	46945.66	949.76
3	52360.22	46244.48	950.65
4	50313.59	44279.51	879.26
5	50411.55	44620.42	943.10
6	50941.39	45676.68	958.00
7	50518.97	46025.28	946.40
8	51254.82	46531.96	907.81
9	50896.88	47543.96	917.37
10	49788.66	47604.71	1064.58
11	51091.62	49058.47	944.93
12	49251.94	47800.80	1074.74
13	48323.10	47011.39	964.11
14	47746.59	47016.20	941.40
15	50093.51	50998.89	973.83
16	48458.34	50110.92	918.24
17	47746.58	49305.93	817.64
18	47237.37	49573.26	831.48
19	49510.46	51574.21	895.60
20	48804.71	51540.33	880.00

The resulted equations consist of ten unknowns. These are three rotations (ω, φ, κ) which are implicit in m's, three translations (X_L, Y_L, Z_L) which represent the camera principal center coordinates with respect to the object space coordinates system, two scale factors (f_x, f_y) and two shifts (x_p, y_p) of the principal point coordinates with respect to the arbitrary coordinates system (x, y). The solution for the unknowns in Eq. 6 requires at least 5 control points as in Burnside (1988).

Space resection: Space resection is the process used to compute the space position (X_L, Y_L, Z_L) and the orientation of the photograph based on ground control points that can be identified on the photograph (Wolf, 1983).

Space intersection: A single photograph is a two-dimensional representation of a three-dimensional scene. Therefore, it is not sufficient enough to locate the three-dimensional positions of ground objects (Methley, 1987). The overlapping photographs then define two intersecting rays to each object point. The intersection of these conjugate rays will determine the object coordinates. The computational process used to resolve such a situation is termed space intersection (Kilford *et al.*, 1979).

MATERIALS AND METHODS

Based on an aerial stereo-pair (60% overlap) acquired by an aerial camera of 152.77 mm focal length, 23×23 cm format, 20 image points of a known X, Y and Z ground coordinates were identified. Eight well-distributed points out of these points were selected as control points (Table 1).

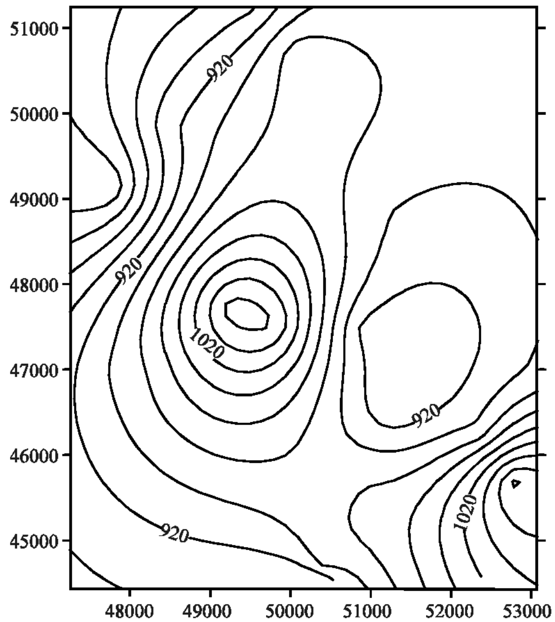

Fig. 1: Contour map of the actual coordinates

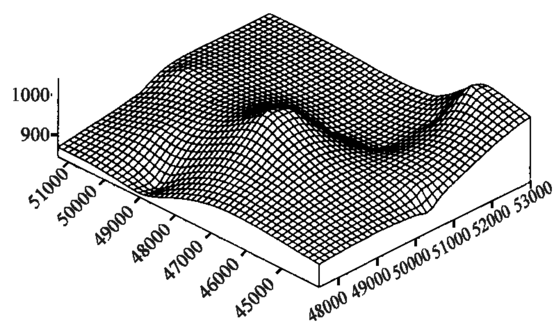

Fig. 2: Three-dimensional representation derived from the computed coordinates based on the 6-terms collinearity equations

Figure 1 represents a contour map derived from the actual coordinates of all points.

A set of measurements for x and y photo-coordinates was carried out for all points using AVIOLYT BC2 with measuring accuracy of 1 μm (Table 2).

Based on the eight control points least squires technique were used for the solution of the 6- parameters of the collinearity condition equations (Burnside, 1988).

Having computed the exterior orientation elements for the left and the right photographs, the ground coordinates (X, Y, Z) for all other points on the stereo-pair were computed (Table 3).

Figure 2 demonstrates a three-dimensional representation derived from the computed coordinates of all points (Table 3).

Table 2: Measured photo coordinates

	Photo coordinates (mm)			
	Left photo		Right photo	
Point	x	y	x	y
1	20.930	90.192	-69.454	86.330
2	-3.730	67.062	-91.779	63.162
3	20.605	70.314	-67.728	66.635
4	87.206	79.012	-0.668	76.006
5	81.377	74.530	-7.309	71.444
6	56.293	61.537	-32.357	58.242
7	59.524	48.706	-28.887	45.497
8	37.821	48.873	-19.901	45.484
9	30.651	24.330	-56.933	20.971
10	52.476	7.839	-37.240	4.641
11	5.724	-2.273	-81.858	-5.736
12	60.405	-3.720	-29.413	-6.828
13	88.696	-1.363	0.281	-4.199
14	99.552	-9.547	11.474	-12.277
15	-2.144	-54.212	-89.673	-57.535
16	41.975	-59.471	-45.088	-62.489
17	66.016	-53.037	-19.864	-55.873
18	72.211	-65.297	-13.789	-68.088
19	1.122	-72.743	-85.213	-75.945
20	15.161	-81.747	-71.017	-84.845

Table 3: Result of the first test

	Computed space coordinates (m)		
Point	X	Y	Z
1	52802.05	45638.11	1082.59
2	53103.52	46944.70	947.50
3	52360.08	46242.75	948.20
4	50313.59	44279.51	879.26
5	50410.89	44619.87	942.77
6	50941.47	45675.57	956.10
7	50518.73	46024.43	945.65
8	51255.11	46530.90	905.72
9	50896.54	47543.47	915.43
10	49788.20	47604.34	1063.67
11	51091.14	49058.30	944.18
12	49251.94	47799.92	1072.92
13	48322.30	47010.71	963.38
14	47744.94	47014.84	937.92
15	50093.69	50999.12	974.78
16	48456.74	50111.49	915.97
17	47743.91	49305.72	812.48
18	47232.16	49575.00	822.45
19	49510.46	51574.21	895.60
20	48804.35	51540.97	879.28

The 10-parameters of the collinearity condition equations were computed using the same eight control points used in the first test. Then by using back intersection (described above) the ground coordinates of the other points were computed (Table 4). Figure 3 demonstrates a three-dimensional representation derived from the computed coordinates of all points (Table 4).

RESULTS AND DISCUSSION

Referring to Table 1 and 2, the results of 6-terms and of 10-terms of the collinearity condition equations, are relatively equal. This can be evaluated by computing the standard error of the computed coordinates for each method of solution (Table 5).

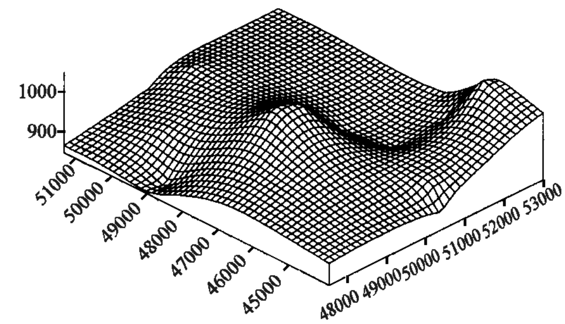

Fig. 3: Three-dimensional representation derived from the computed coordinates based on the 10-terms collinearity equations

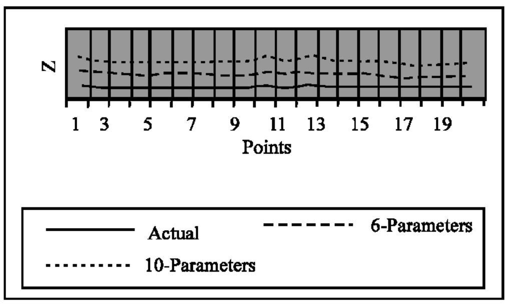

Fig. 4: Graphical representation of actual and computed height (Z)

Table 4: Result of the second test

Point	Computed space coordinates (m) X	Y	Z
1	52804.02	45639.61	1084.07
2	53102.61	46944.44	947.18
3	52360.18	46242.34	946.79
4	50313.59	44279.51	879.26
5	50411.75	44623.81	949.46
6	50942.01	45676.52	957.69
7	50519.27	46025.42	947.43
8	51255.00	46531.56	904.01
9	50896.94	47543.61	914.48
10	49789.09	47605.25	1062.46
11	51091.07	49058.68	943.25
12	49252.00	47800.78	1069.05
13	48323.52	47011.92	965.85
14	47746.59	47016.20	941.40
15	50093.25	51000.67	972.15
16	48455.88	50112.88	913.09
17	47748.10	49305.42	822.20
18	47236.66	49573.52	831.34
19	49510.46	51574.21	895.60
20	48804.64	51540.73	879.39

Table 5: Standard errors of the tests results

Coordinates	Standard error (m) 6-trems	10-terms
X	±1.44	±0.82
Y	±0.93	±1.11
XY	±1.66	±1.45
Z	±2.88	±2.94
XYZ	±3.35	±3.25

From Table 5, it can be seen that the planimetric precision of the first and the second tests were found to be 1.66 m and 1.45 m, respectively which meat the topographic mapping specifications at all scales.

For the height Z the standard error was 1.66 m for the first test and 1.45 m for the second test. Figure 4 represents the actual and computed height of points.

Close results were fond in linear XYZ standard error for the first and second test; these are 3.35 and 3.25 m, respectively.

CONCLUSION

The derived three-dimensional space coordinate, using collinearity condition equations with 6-terms is relatively equal to that derived using 10-terms. This may be due to the fact that the metric camera is a high precision and minimum distortion. Therefore, it is economical and sufficiently accurate to use 6-terms collinearity conditions equation in aerial photogrammetric topographic mapping.

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