



## ORIGINAL ARTICLES

### Development of Predictive Markov-chain Condition – Based Tractor Failure Analysis Algorithm

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#### ABSTRACT

A repairable mechanical system (as agricultural tractor) is subject to deterioration or repeated failure. The system is subjected to periodic inspection that identifies the condition of deterioration. Based on the degree of deterioration (system condition), preventive maintenance is performed or no action is taken. At each inspection of failure the system status is classified into partial, Combined and complete. According to this Condition-Based Maintenance classification the level of maintenance is determined and performed to restore the system to "as good as new" state. The level of maintenance is either general repair (minor replacement of parts by preventive maintenance) or complete overhaul (major replacement of parts by corrective maintenance). Condition-Based Maintenance (CBM) is a methodology that strives to identify incipient faults before they become critical to enable more accurate planning of preventive actions. For the ultimate success of CBM methodology, we must have sound methods for modeling deterioration; quantify their conditions and effects, and identify the optimal selection and scheduling of preventive maintenance actions (the right action at the right time). Considerable research has been conducted on this issue of periodic replacement times of failing system. No doubt this is a reflection, at least in part of the high capital cost of many farming equipments and the importance of minimizing unnecessary failure costs. Despite the relatively the large body of literature on this topic, analysis of dynamic maintenance schedule and their effect on performance of the system, remains as an open problem. To fill the gap, this paper presents a tractor condition-based maintenance model for a deteriorating system using a simple recursive Markov chain closed-form analytical solution. The model of tractor failure analysis is based on type of failure and tractor operating hours on the field by expressing in mathematical forms the optimum preventive and corrective maintenance and forecast future failures. The aim of building analytical, user- friendly algorithm is to increase reliability and availability of tractor life time cycle, decreased cost of repair and maintenance and aid farm managers, agricultural engineers and decision-makers to formulate maintenance and system/equipment replacement strategies. Hence, six performances measures of the model process are used to find the optimal algorithm parameters that maximize the system availability. The model decision variables are working hours, time to repair and number of failure. Thereafter, the model is used to predict the expected number of failure (number of CM) by sorting equation data in the Microsoft Excel operating environment to build tables of failure times and inter- failure times and availability based on user – defined selection of data from the model database. To statistically verify model accuracy its basic functional relations (decision state matrix) are compared with Amari and Mc Langhim model (2004) and WINQSB software (Version 1.00) using field data from tractors working in Sugar cane plantations in Sudan.

**Key words:** Markov-chain; Failure analysis; Condition-based maintenance; Transition probability; Reliability; Tractor maintenance and replacement strategies.

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#### Introduction

Worldwide Tractors is the main source of power in the farm, and represents a major component of farm fixed costs. If properly and given the necessary field maintenance tractors will operate for long period and do

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a great deal of work before major repairs are required (Elbashir, 1996). Tractor break down can be of high cost not only from the stand point of the expenditure necessary to effect repair, but also because of the disastrous effect on crop productivity and the fact that idle staff must still be paid. The extent of the problem of tractor failure in developing countries is more serious compared with developed countries. This is due to acute shortages in genuine spare parts, affecting preventive maintenance, absence of future planning for integrated maintenance management and programs that strive to identify incipient faults before they become critical to enable more accurate planning of preventive actions. As such system performance can be improved by developing optimal maintenance prediction models that minimize overall maintenance cost or maximize system performance measures. Maintenance is a complex process and its functions as defined by Laskiewicz, (2005) include: Inspection, verification, and correction operations. Bowler, (1999) defined two type of tractor maintenance: Preventive maintenance (PM) conducted to keep equipment working and/or extend the life of the equipment, Corrective maintenance (CM), sometimes called "repair", conducted to get equipment working again. However, the sophistication of maintenance models has increased with the growth in the complexity of modern systems of agricultural tractors (Valdez-Flores and Feldman, 1989), which in turn has increased the complexity of the analysis and solution procedures. Therefore, many practitioners as well as some researchers (Rao and Bhadury, (2000) have turned to the simulation approach to solve these problems. In general, simulation requires more computational time to achieve the desired accuracy. Because we cannot obtain the closed-form solution with simulation, we can only use "black-box" optimization methods to find the optimal maintenance parameters. Therefore, it is important to solve the problem analytically whenever possible, and use other methods such as simulation only when analytical solutions are not possible or computationally expensive. One of pioneer work is that of Liu, (1991) who defined A Markov process as stochastic process, usually, visits more than one states and it is a finite set. He also stated that an important property of a Markov process is that it jumps regularly, it jumps after unit time, so the system either transition (moves) to a new state or else the system returns to the current state. Barbera, (1999) investigate the use of Hidden Semi-Markov Model (HSMM) to predict the remaining useful life (RUL) on maintenance and predict reliably of components. Amari and McLaughlin, (2004) studied System performance and associated gain to improve by using efficient maintenance policies by Markov-chain. They reported that the latest developments in maintenance models include condition based maintenance strategies. As cited by Amari (2004). Gertshbakh. *et al* (1996), discussed some modeling errors and limitations in the existing maintenance models. They reported that because of the complexity of these models, it is very difficult to solve them analytically. Therefore, using automatic conversion to Markov chains, only numerical solutions are presented in. However, numerical methods are inefficient to study the behavior of the system in detail and cannot be used with efficient optimization methods. In order to overcome this problem, the main objective of this study is to develop a predictive Markov -chain condition – based tractor failure analysis model. Hence, in this study, Markov analysis is used to provide and describe a closed-form analytical solution for a simplified maintenance model. Then the algorithm is planned to finds the optimal model parameters that maximize the availability of the system. The model shall be statistically verified by studying its theoretical basic characteristics in reference to the model of Amri and McLanghim (2004) and in comparison with WINQSB software using field data from tractors working in Sugar cane plantations in Sudan.

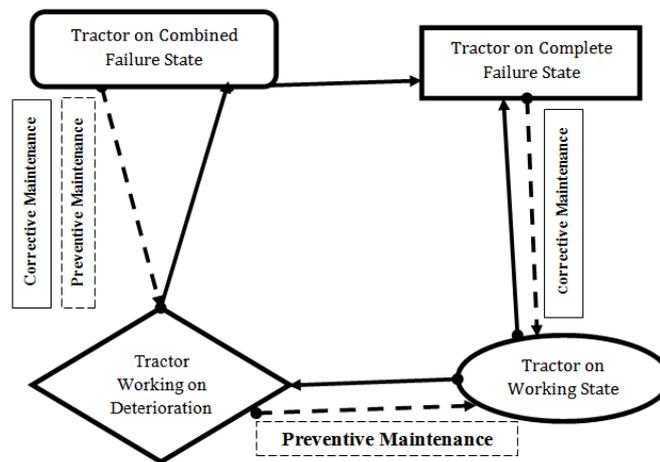
## Materials and Methods

### *Model Development:*

The model of tractor failure analysis is based on type of failure and tractor operating hours on the field. Failure rate is usually time dependent, and an intuitive corollary is that both rates change over time versus the expected life cycle of a system. So, the failures occur during the period of tractor operation can be categorizing to: Partial failure state, combined failure state and complete failure state (Figure.1).

Failure rate is defined as the frequency with an engineered system or component fails, expressed for example in failures per hour (It is often denoted by the Greek letter  $\lambda$  lambda) and is important in reliability theory). In practice, the reciprocal rate of maximum time before failure (MTBF) is more commonly expressed and used for high quality components or systems as given by Mishera, (2006).

According to Ishola, (2004) tractor general components (machine, hydraulics system, transmission system, electric system and steering break system) and the complete list of all possible states can be simulated as block transition diagram by using the general complete system model of tractor failure states with two type of maintenances. If the failure is donated as  $\lambda$  and maintenance is donated as  $\mu$ , so we can describe the transmissions state before spaces of system model, as:



**Fig. 1:** General Complete System Models of Tractor Failure States

1.  $S(A, 0)$  tractor working good as new and  $\lambda \leq t \geq 1$  where:  $t$  is operating, hour, and  $A$  is state 0;  $S(a_1, 0)$   $\lambda \leq t \geq 1$
  3.  $S(a_1, a_2)$  tractor on complete failure  $\mu \leq t \geq 1$   $a_2$  is state order, so the tractor must be under corrective maintenance as  $S(a_2, 0)$   $\lambda \leq t \geq 1$
  4.  $S(A, e_1)$  tractor is operating in the other deterioration stage and  $\lambda \leq t \geq \mu$  and  $e_1$  is state order;  $S(e_1, 0)$   $\lambda \leq t \geq 1$
  5.  $S(e_1, e_2)$  tractor on complete failure  $\mu \leq t \geq 1$   $e_2$  is state order, so the tractor must be under corrective maintenance as:  $S(e_2, 0)$   $\lambda \leq t \geq 1$
  6.  $S(a_1.e_2)$  tractor on combined failure state and it is under inspection after the preventive maintenance threshold deterioration stage;  $\mu \leq t \geq \lambda$
  7.  $S(a_2.e_2)$  tractor on combined failure state and it is under inspection after the preventive maintenance threshold deterioration stage;  $\mu \leq t \geq \lambda$
  8.  $S(a_2.e_1)$  tractor on combined failure state and it is under inspection.
  9.  $S(A, b_2)$  tractor on complete failure  $\mu \leq t \geq \lambda$
  10.  $S(A, c_2)$  tractor on complete failure  $\mu \leq t \geq \lambda$
  11.  $S(A, d_2)$  tractor on complete failure  $\lambda \leq t \geq 1$
  12.  $S(A, a_2)$  tractor on complete failure  $\mu \leq t \geq 1$
- $a_2$  is state order, so the tractor must be under corrective maintenance as  $S(A, f_2)$   $\lambda \leq t \geq 1$

Table 1 shows the complete states of failure for tractor system with description example for any stage while the figure 2 shows the availability transition rate of failure and repair before operation states and different failure states.



$$S^{a_2}(t) = \lambda_{a_1}^A \cdot (t) + \lambda_{a_2}^A \cdot (t) \quad t_A \leq t \leq t_{a_2} \tag{3}$$

Other states is direct failure can be written as:

$$S^{b_2}(t) = \lambda_{b_2}^A \cdot (t) \quad t_A \leq t \leq t_{b_2} \tag{4}$$

Tractor on Combined Failure State ( $a_1e_2, a_2e_1, a_2e_2$ ):

Failure rate of the states ( $a_1e_2, a_2e_1$ ) is same is transition but it's different on duration:

$$S^{a_1e_2}(t) = \varepsilon_{a_1e_2}^A \cdot (t) \quad t_A \leq t \leq t_{a_1e_2} \tag{5}$$

Failure transition rate is the frequency with which tractor system or component fails, failure rate is usually time dependent so to obtain the interval rate:

$$\lambda = S_{A_t}, S_{A_{t+1}}, S_{A_{t+2}} \dots, S_{A_{t+i-1}}, S_{a_{n_t}} \tag{6}$$

Where:  $S_{A_t}$  = First step of transition rate,  $S_{A_{t+1}}$  = Second step of transition rate,  $S_{A_{t+i-1}}$  = Step before last of transition rate (where tractor is faulty),

$S_{a_{n_t}}$  = Last step of transition rate (where tractor is failed).

The transition rate of failure can be estimated as (MacDiarmid, *et al*, 1998.) equation:

$$\lambda = \frac{\text{Number of failure}}{\text{working hours}} \tag{7}$$

So, the maximum time before state start failure can be calculate by limit integral of sate equation before first step and before last as

$$S^{\max}(t) = \int_{A_t}^{A_{t+i-1}} \frac{n}{t_h} \cdot dt \tag{8}$$

Where n Number of failure  $t_h$  Working hours. Moreover, we have:

$$S^{\max}(t) = [n \cdot \ln(t)]_{A_t}^{A_{t+i-1}} \tag{9}$$

By solving equation for i, we obtain the maximum time before fault (time to inspection and made PM)

$$S^{\max}(t) = (n \cdot \ln(t+i-1)) - (n \cdot \ln(t)) \tag{10}$$

$$S^{\max}(t) = n \cdot \left[ \ln\left(\frac{t+i-1}{t}\right) \right] \tag{11}$$

The probability of step before failure (t+i-1) estimation by Markov chain which is discussed later, and by applying equation (11) to all failure states we obtain:

- Tractor on Partial Failure State ( $a_i, e_i$ )

$$S^{a_i \max}(t) = n \cdot \left[ \ln\left(\frac{a_{1t+i-1}}{A_t}\right) \right] \tag{12}$$

-Tractor on Complete Failure State ( $a_2, e_2, b_2, c_2, d_2, f_2$ )

$$S^{a_{2\max}}(t) = n \cdot \left[ \ln \left( \frac{a_{2t+i-1}}{A_t} \right) \right] \tag{13}$$

-Tractor on Combined Failure State ( $a_1e_2, a_2e_1, a_2e_2$ )

$$S^{a_1e_{2\max}}(t) = n \cdot \left[ \ln \left( \frac{a_1e_{2t+i-1}}{A_t} \right) \right] \tag{140}$$

The state of tractor operation represent as probability transition rate of Markov-chain which can be defined by the following equation:

$$P = (X_n = i, X_{n+1} = j, X_{n-1} = i_{n-1}) \cdots X_n = i_1 \tag{15}$$

Markov chain analysis as described by Barbera, (1999) looks at a sequence of event, defined as transitions before states, and calculates the relative probability of encountering these events in both the short-run and the long-run. Markov chain is useful for analyzing dependent random failure, which is failure whose likelihood depends on what happened currently. So the probability formula of Markov chain with rearrangement is:

$$P = (X_n = i, X_{n+1} = j) \tag{16}$$

Transition probability before tractor failure states for one cycle life by using Markov formula (Homogenous Markov chain) on one-step transition probability is billed as:

$$P_{n,n+1} = P_{ij} \tag{17}$$

Hence, the approximation of getting probability observation with a system becomes:

$$P_{ij} = \begin{cases} \frac{\lambda + i}{n}, & j = i + 1 \ (i = 0, 1, 2, 3), \\ \frac{\lambda_2 + i}{n}, & j = i + 2 \ (i = 1, 2, 3, 4), \\ \frac{\lambda + i}{n}, \frac{\lambda_2 + i}{n} & j = i \ (i = 1, 2, 3, 4) \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

Where:

$$\frac{\lambda + i}{n} = \text{Probability of partial failure transition rate,}$$

$$\frac{\lambda_2 + i}{n} = \text{Probability of complete failure transition rate,}$$

$$\frac{\lambda + i}{n}, \frac{\lambda_2 + i}{n} = \text{Probability of combine failure transition rate,}$$

0 = Tractor on operation state

For one cycle life of tractor operation, homogenous Markov chain includes failure transition and repair transition rate which is symmetrical failure rate on probability:

$$P_{ij} = \begin{cases} \frac{\mu-i}{n}, & j = i-1 \ (i = n, n-1, n-2, n-3), \\ \frac{\mu_2-i}{n}, & j = i-2 \ (i = 1, 2, 3, 4), \\ \frac{\mu-i}{n}, \frac{\mu_2-i}{n}, & j = i \ (i = n) \end{cases} \quad (19)$$

Markov chain Probability equation used to calculate probability transition of failure on different states:

$$P_i^A = \frac{\lambda_i^A}{\mu_i^A + c} \quad (20)$$

Where:

$P_i^A$  = Probability transition before state A (operation) and state i (fault or failure),

$\lambda_i^A$  = Failure transition Rate before States,

$\mu_i^A$  = Repair transition Rate before States (A) and i,

C = Common Failure Rate (depend on state), on partial state C donate as  $\epsilon$  and calculate by the following equation:

$$\epsilon = \lambda_i^A + \lambda_{i+1}^A \quad (21)$$

The over all probability of tractor life cycle calculated by summation the probabilities states

$$P^A = \sum_{i=n}^n \left[ 1 + \frac{\lambda_i^A}{\mu_i^A + C} \right]^{-1} \quad (22)$$

By using equation 35, 37 we obtain Partial state probability as follows:

*-Partial Failure State*

Before A and  $a_1$ :  $P_{a_1}^A = \frac{\lambda_{a_1}^A}{\mu_{a_1}^A + \epsilon} \cdot P^A \quad (23)$

Naturally this rate is independent of (n+1,0), because the rate is, by definition, a measure of the propensity to enter a particular state for entities that are not presently in that state, which is clearly independent of the rate of leaving that state. Interestingly, if we define A-  $e_1$  as infinite for each j, none of the state equations are affected, because the infinite "self-transition" flow  $P^A e_1$  is both added to and subtracted from the al equation, but this enables us to write the equation for the failure rate in the more unified form.

*-Complete state probability:*

Before A and  $a_2$ :  $P_{a_2}^A = \frac{\lambda_{a_1}^A \cdot \lambda_{a_2}^A}{(\mu_{a_1}^A + \epsilon) \cdot \mu_{a_2}^A} \cdot P^A \quad (24)$

*-Combined state probability:*

Before A and  $a_1 e_2$ :  $P_{a_1 e_2}^A = \frac{\lambda_{e_1}^A \cdot \epsilon}{(\mu_{e_1}^A + \epsilon) \cdot \psi} \cdot P^A \quad (25)$

By recalculating Common Failure Rate (C) as  $\Psi$  and  $\delta$

$$\delta = \lambda_{a_2}^A + \lambda_{a_2}^A \quad (26)$$

$$\rho = \mu_{a_2}^A + \mu_{a_2}^A \quad (27)$$

By using obtained result from Markov chain Probability the maximum time before failure (MTBF) can be estimate as:

$$S^{\max}(t) = n \cdot \left[ \ln \left( \frac{1 - P_i^A}{\lambda_i} \right) \right] \quad (28)$$

The application of equation 52 to all system can be summarized as:

*-Partial failure state*

$$S^{a_1 \max}(t) = \left| n \cdot \left[ \ln \left( \frac{1 - P_{a_1}^A}{\lambda_{a_1}} \right) \right] \right| \quad (29)$$

*-Complete failure state:*

$$S^{a_2 \max}(t) = \left| n \cdot \left[ \ln \left( \frac{1 - P_{a_2}^A}{\lambda_{a_2}} \right) \right] \right| \quad (30)$$

*-Combined failure state:*

$$S^{a_2 e_1 \max}(t) = \left| n \cdot \left[ \ln \left( \frac{1 - P_{a_2 e_1}^A}{\lambda_{a_2 e_1}} \right) \right] \right| \quad (31)$$

Figure 3 shows the flow chart of failure analysis model. The selection of model is based on understanding the failure behavior of the repairable tractor, by providing a mathematical equation to optimize the preventive and corrective maintenance. The model forecast future failures through the mathematical formulation and also optimize maintenance strategy for the repairable system by analyzing the relevant information. From the stochastic point of view of the process it is also important to determine the process failure trend, to know whether a failure rate is increasing, decreasing or constant. One of the most important decisions that a maintenance manager must take comprises the timing of system/equipment replacement. This is to be done to balance optimally before the frequencies of maintenance against the expected failure. Various failures were identified and categorized as mechanical to complete, combined and partial. After the model is formulated and its parameters estimated, it can be used to predict the expected number of failure (number of CM) by sorting equation data in the Microsoft Excel operating environment to build tables of failure times and inter- failure times and availability based on user – defined selection of data from the model database.

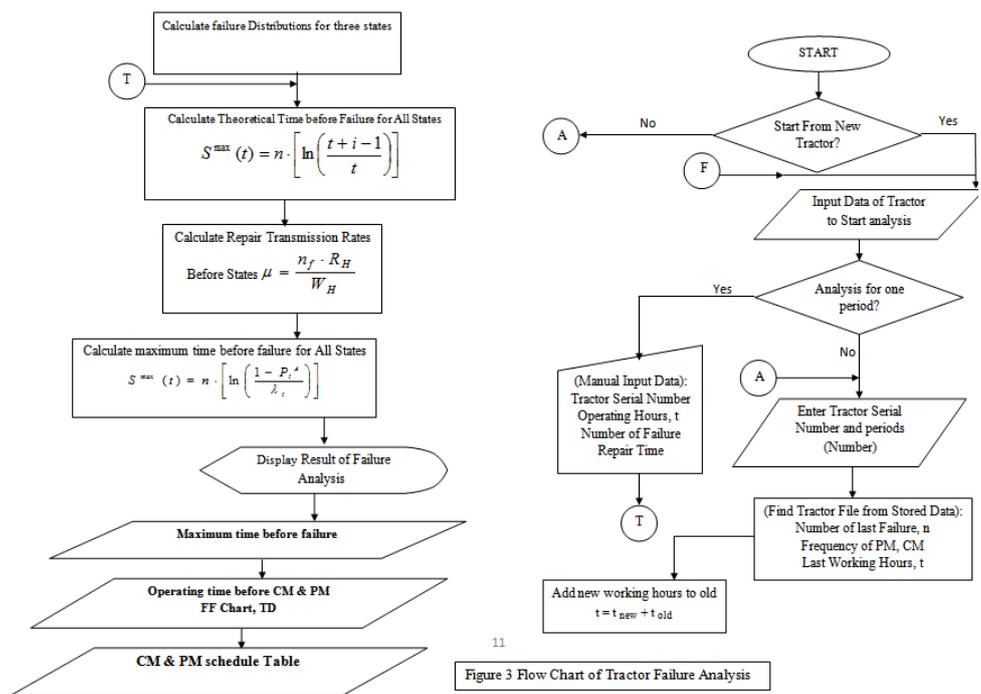
*Model Output:*

The output of model includes:

1- The probability of transition rate (before all passable states, for one cycle operation life of tractor): The equation of Markov chain probability transition rate summarized on table 2

**Table 2:** Probability of transition Rate of Failure and Repair

MODEL PROBABILITY	$\sum_{i=1}^n \left[ 1 + \frac{\lambda_i^A}{\mu_i^A + C} \right]^{-1}$
MODEL AVAILABILITY	$P^A + P_{e_1}^A + P_{a_1}^A$
FREQUENCY of a1 to A	$P_{a_1}^A + \mu_{a_1}^A$
FREQUENCY of a2 to A	$P_{a_2}^A + \mu_{a_2}^A$
FREQUENCY of e1 to A	$P_{e_1}^A + \mu_{e_1}^A$
FREQUENCY of e2 to A	$P_{e_2}^A + \mu_{e_2}^A$
FREQUENCY of b2 to A	$P_{b_2}^A + \mu_{b_2}^A$
FREQUENCY of c2 to A	$P_{c_2}^A + \mu_{c_2}^A$
FREQUENCY of d2 to A	$P_{d_2}^A + \mu_{d_2}^A$
FREQUENCY of f2 to A	$P_{f_2}^A + \mu_{f_2}^A$
FREQUENCY of a2e1 to A	$P_{a_2e_1}^A + \mu_{a_2e_1}^A$
FREQUENCY of a1e2 to A	$P_{a_1e_2}^A + \mu_{a_1e_2}^A$
FREQUENCY of a2e2 A	$P_{a_2e_2}^A + \mu_{a_2e_2}^A$



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Figure 3 Flow Chart of Tractor Failure Analysis

**Fig. 3:** Flow Chart of Tractor Failure Analysis

The above Markov Chain output can be transferred to maintenance information namely failure frequency between different states by estimating failure analysis as shown on the table 3:

**Table 3:** failure frequency and maintenance policy between different states.

CM&PM DURATION	
Mean of PM	Inverse of FREQUENCY of a <sub>1</sub> to A
Mean of CM & PM	Inverse of FREQUENCY of a <sub>2</sub> to A
Mean of PM	Inverse of FREQUENCY of e <sub>1</sub> to A
Mean of PM &CM	Inverse of FREQUENCY of e <sub>2</sub> to A
Mean of CM	Inverse of FREQUENCY of b <sub>2</sub> to A
Mean of CM	Inverse of FREQUENCY of c <sub>2</sub> to A
Mean of CM	Inverse of FREQUENCY of d <sub>2</sub> to A
Mean of CM	Inverse of FREQUENCY of f <sub>2</sub> to A
Mean of PM &CM	Inverse of FREQUENCY of a <sub>2</sub> e <sub>1</sub> to A
Mean of PM &CM	Inverse of FREQUENCY of a <sub>1</sub> e <sub>2</sub> to A
Mean of PM &CM	Inverse of FREQUENCY of a <sub>2</sub> e <sub>2</sub> to A
Total Number of PM	Summation of Mean of PM
Total Number of CM	Summation of Mean of CM

2- Prediction of Maintenance Policy:

This outputs describe the forecasting of failure behavior for the next time according to last operating hours and number of failure during this time, its includes:

1- Maximum time before failure; which can be calculated as follow:

$$MTBF = \left| n \cdot \left[ \frac{\ln(1 - p_i^A)}{P^A} \right] \right| \tag{32}$$

Where:  $P^A$  = System Probability from Markov chain calculates

This time can be shown graphically through the three states of failure. The graph can be helpful tools to the decision maker to know which type of machine recorded high time before failure and link that with filed operation condition.

3- Tractor availability:

Availability prediction and assessment methods can provide quantitative performance measures that may be used in assessing a given design or to compare system alternatives to reduce life cycle costs. This technique increases the probability of mission success by ensuring operational readiness. Analyses based on availability predictions will help assess design options and can lead to definition of maintenance support concepts that will increase future system availability; anticipate logistics and maintenance resource needs. It is calculated as:

$$TV = \sum_{i=1}^n P_i^A \tag{33}$$

4- Tractor Dependability:

By using MTBF the tractor dependability which is obviously a desirable system attributes and even if a system is designed to be "dependable," it is likely that it will need maintenance at some point in its life; TD can be obtained from the equation:

$$\frac{MTBF}{(MTBF + MTTR)} \cdot 100 \tag{34}$$

5- Operating time before PM and CP:

This term is illustrated from markov chain probability outputs namely model probability. It is calculated as transition probabilities of all states (partial combine and complete). Operating time before PM and CP is indicator tools for how downtime can be reduced as percentage of probability transition before states if using optimal predict of maintenance policies.

The following equation is used to calculate CM:

$$CM = \frac{\sum_{i=1} t_i}{\sum_{i=1} CM_i} \tag{35}$$

Where:  $t_i$  = Duration of operating during specific states (probability transition state)

CM = Preventive maintenance on state i. Preventive maintenance calculates by the same equation but at different probability transition state.

**Results and Discussion**

*Model Verification and Validation:*

The failure analysis model for management of tractor maintenance is verified and validated for purposes of testing model correctness, effectiveness, satisfaction of purpose, and reflecting model internal processes, structure and buildup. The evaluation process made at two levels: namely, verification and validation. The testing of model verification was made by examining the building of model internal structure using: i) Amari and McLaughlin, (2004) for testing Maximum time before failure (MTBF) and ii) (WinQSB) Software Program of Markov Process Model (version 1.00) to test probability transition matrix before states using descriptive statistical tools. While model validation was examined by comparing performance of the developed model with that generated from field data as described by Macal, (2005).

i) *Comparison with Amari, and McLaughlin, (2004) model:*

This test is used to evaluate the knowledge base of the equations used as integral base of the structure of the developed model that used to calculate the maximum time before failure. The equation used by the developed model originates from Markov chain probability while the one employed by Amari, and McLaughlin (2004), is founded on the general Markov process defined by Schneider, (1998) and Bello (2006).The two equations written for the developed model and for Amari, and McLaughlin (2004), and McLaughlin (2004) model respectively are as follows:

$$P = (X_n = i, X_{n+1} = j)$$

$$P = \sum_{i \in S} (P\{T \leq n | X_0 = i\} P\{X_0 = j\})$$

Where: i state at present time and j state at next time, n number of states, T is function limits of states (integer number). Each model use it is basic equation to develop the probability of transition matrix for each one of the three failure states (partial, combined and complete).The model assumptions can be satisfied with the above parameters at three states (partial, combined and complete) by the mathematical formula of one-step transition probability before these states as follows:

$$p_{ij} = \begin{cases} \frac{\lambda + i}{n}, & j = i + 1 \ (i = 0, 1, 2, 3), \\ \frac{\lambda_2 + i}{n}, & j = i + 2 \ (i = 1, 2, 3, 4), \\ \frac{\lambda + i}{n}, \frac{\lambda_2 + i}{n} & j = i \ (i = 1, 2, 3, 4) \\ 0, & \text{otherwise} \end{cases}$$

Likewise, the probability compatible with the example presented by Amari, and McLaughlin (2004), for probability before three states of transition matrix is defined using the following conditional expressions:

$$P_{ij} \begin{cases} 1 \leq i \leq n, & \text{no, maintenance, is, performed} \\ n \leq i \leq k, & \text{preventive maintenance}(k), \text{is, performed,} \\ i, 1 \leq i \leq k & \text{system is operating in the deterioration stage} \\ 0, & \text{otherwise} \end{cases}$$

The transition matrix of above probabilities applies for all possible combinations by the two solution systems. Following tables (table 4 and table 5) shows the theoretical matrix of development model and Amari, and McLaughlin (2004) model:

**Table 4:** Transition Matrix of Development Model

State	Partial	Combined	Complete
Partial	$\frac{\lambda + i}{n}$ ,	$\frac{\lambda + i}{n}$ ,	$\frac{\lambda + i}{n}$ ,
Combined	$\frac{\lambda + i}{n}, \frac{\lambda_2 + i}{n}$	$\frac{\lambda + i}{n}, \frac{\lambda_2 + i}{n}$	$\frac{\lambda + i}{n}, \frac{\lambda_2 + i}{n}$
Complete	$\frac{\lambda_2 + i}{n}$	$\frac{\lambda_2 + i}{n}$	$\frac{\lambda_2 + i}{n}$

**Table 5:** Transition Matrix of Amari and McLaughlin (2004) model

	Partial	Combined	Complete
Partial	$p(i, 0) \cdot \frac{\lambda}{n}$ ,	$p(i, 0) \frac{\lambda}{n}$ ,	$p(i, n) \frac{\lambda}{n}$ ,
Combined	$p(i, 0) \frac{\lambda}{n}, \frac{\lambda_2}{n}$	$p(i, n) \frac{\lambda}{n}, \frac{\lambda_2}{n}$	$p(i, k) \frac{\lambda}{n}, \frac{\lambda_2}{n}$
Complete	$p(I, i), \frac{\lambda_2}{n}$	$p(I, n), \frac{\lambda_2}{n}$	$p(I, k), \frac{\lambda_2}{n}$

It's clear from two tables no different in result of transition matrix before three states, so for example of partial state:

1- Transition matrix before Partial states to Partial states for development model:

$$\frac{\lambda + i}{n} = \frac{\lambda_i}{n} + \frac{\lambda_0}{n} = \frac{\lambda_i}{n}$$

2- Transition matrix before Partial states to Partial states for Amari, and McLaughlin (2004) model:

$$p(i, 0) \frac{\lambda}{n} = 1 \cdot \frac{\lambda}{n} = \frac{\lambda}{n}$$

Finally to calculate the maximum time before failure for two models used the following equations:

1- For development model:

$$S^{\max}(t) = n \cdot \ln\left(\frac{t + \lambda - 1}{t}\right)$$

By expanding the equation:

$$n \cdot \ln\left(\frac{t + \lambda - 1}{t}\right) = n \cdot \ln(t + \lambda - 1) - n \cdot \ln(t)$$

If t = 1 (for complete time):

$$n \cdot \ln(1 + \lambda - 1) - n \cdot \ln(1) = n \cdot \ln(\lambda)$$

2- For Amari, and McLaughlin (2004) model:

$$S^{\max}(i, t) = \frac{1}{P(n, i) \cdot \lambda}$$

$$S^{\max}(i, t) = \frac{1}{P(n, i) \cdot \lambda} = \frac{1}{\lambda} - n \frac{1}{\lambda}$$

By integral for complete time:

$$\int_i^t S^{\max}(i, t) dt = \int_i^t n \cdot \frac{1}{\lambda} d\lambda$$

$$p(i, 0) \frac{\lambda}{n}, = 1 \cdot \frac{\lambda}{n} = \frac{\lambda}{n}$$

Finally:  $S^{\max} = ([n \cdot \ln(\lambda)]_i^t$

The maximum time before failure is typical to both model development and Amari, McLaughlin (2004) model. Hence, it is evident from the above discussion that the formulations of basic equations for building the structure of the two models results in similar outputs.

ii) Comparison with WinQSB Software:

By using the WinQSB program of Markov Process for probability transition matrix before three states (Partial, combine, and complete) with probability transition matrix of developed model the probability output shows in table (6).

**Table 6:** Probability Output of Developed Model and WinQSB Program

States	Partial		Combine			Complete			P			
$P_{ij}$	$a_i$	$e_i$	$a_i e_i$	$a_i e_i$	$a_i e_i$	$a_i$	$e_i$	$b_i$	$c_i$	$d_i$	$f_i$	
Model	0.044	0.014	0.097	0.067	0.085	0.064	0.084	0.097	0.197	0.097	0.097	0.943
WinQSB	0.037	0.013	0.094	0.059	0.084	0.064	0.0821	0.093	0.196	0.093	0.093	0.928

Using T-student-pair comparison test the Probability states of eleven sub-states of failure predicted by the developed model is compared to those generated by WinQSB Markov Process program as shown in Table (7), the table indicates that the differences before the two models vary slightly with respect to each individual sub-state of failure but there is no significant difference with respect to overall probability for all classes of failure.

**Table 7:** T-student-pair comparison Test

Source	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval	
					Lower	Upper
					Model	5.726226
Win Qsb	5.954672	10	0.00014036	0.082554545	0.051664	0.113445077

\*The mean difference is significant at the .05 level

Validation Processes:

As reported by Hillston, (2003) validation may be considered as the task of demonstrating that the model is a reasonable representation of the actual system: that it reproduces system behavior with enough fidelity to satisfy analysis objectives. To arrive to this end the developed model is tested in comparison with field data using the procedure outlined by Macal, (2005).

Data for the year 2006 of Cameco tractor (T220) operating in Sinnar Sugar Factory was used as input data (Table 5.5) to developed model to generate and predict expected average number of failure in the year 2007. As depicted in Table5.6 the model results is compared to actual workshop data for the same tractor in the

same workshop in the year 2007. However, in practice it is difficult to achieve a full validation of the model by running a complete failure analysis (by testing: maximum time before failure (hours), operating time before PM, operating time before CM, and average of frequency) due to measurement problem and data availability and accuracy. For this reason initial validation attempts will concentrate on the main output of the model (the average number of failure), and only if that validation suggests a problem will more detailed validation be undertaken. Table 5.7 indicate that there is no significant difference (at P= 0.05) before the average number of failure predicted by the model and that recorded for the year 2007 for Cameco T220 in Sinnar Sugar Factory.

**Table 8:** Maintenance Data of Cameco Tractor (Season 2006)

MONTH	Wheel Tractor Cameco T 200		
	Working Hours	Num of Failure	Repair Time
MONTH 1	370	32	156
MONTH 2	270	78	144
MONTH 3	472	34	148
MONTH 4	321	57	97
MONTH 5	624	7	128
MONTH 6	502	69	148
MONTH 7	375	47	144
MONTH 8	620	53	110
MONTH 9	555	36	130
MONTH 10	547	22	105
MONTH 11	434	22	127
MONTH 12	616	40	92

**Table 9:** Model Results Compared to Actual Workshop Record Data

Month	Wheel Tractor Cameco T 200	
	Number of failure (2007) from workshop record	Average Number of failure ( Predicted by the model from 2006data)
1	32	34
2	78	72
3	34	33
4	57	56
5	7	29
6	69	68
7	44	49
8	53	65
9	32	39
10	19	26
11	20	27
12	39	45

**Table 10:** Statistical Analysis of average Number of Failure

Failure No.	Mean	Std. Deviation	Std. Error Mean	df	Sig. (2-tailed)
Av No of failure (2007)	-4.916	7.31695	2.112	11	0.04002958
Av No of failure ( 2006)					

\*The mean difference is significant at the .05 level

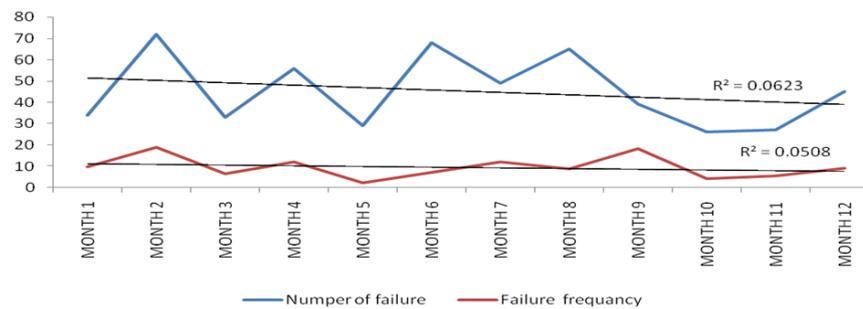
Another testing approach considered from the graph of figure 4 which illustrated relation before numbers of failures from Cameco Tractor on season 2006 compared with predication failure frequency (2007), so it is evident that from this graph the average number of failure trend line has same behavior of predication failure frequency calculated by the model.

From these results it may be inferred that the implemented model is a verified implementation of the assumptions and a valid representation of the real situation.

*Conclusions and Recommendations:*

*Conclusions:*

To obtain a correct failure diagnosis and to prepare more effective maintenance programs, it is essential to have reliable monitoring (inspection) system and forecast the mechanism about the trend of system failure rate.



**Fig. 4:** Numbers of failures from Cameco Tractor (2006) & failure frequency

The developed failure analysis model for management of tractor maintenance proved to be formulated on sound knowledge base and capable enough to predict actual number of failure. These are made firstly by analytical approach to verify the model internal theoretical structure in comparison to Amari, McLaughlin model (2004) and by using statistical approach (t test) in comparison to WinQSB Software for testing probability transition matrix between states and eleven subsystems. Secondly to validate model powers to predict number of failure in relation to actual field using Macal (2005) procedure. Model correctness of the theoretical formulation is achieved by analysis of assumptions and the basic equations used to develop the probability transition matrix for failure states of partial, combined and complete. Also it is made by comparing equations to estimate maximum time before failure and before failure occurs.

#### *Recommendations:*

- 1- The validated and verified model developed in this study is recommended to be an employed for tractors failure analysis and for selecting maintenance policy.
- 2- Selection of suitable tractor type needs to be made with relation to availability quality of maintenance (Workshop quality)
- 3- The model can used to estimate tractor availability factor for adjusting scheduling programme for executing mechanized cultural practices.
- 4-As alternative to the manufacturer maintenance schedule the developed model is recommended to be used to predicted a real and practical programme.
- 5- Analysis of the model may be extended in future to include an additional subroutine for each individual subsystem of the tractors.
- 6-The developed model need to be used for analysis of tractors in the case of un- predictable machinery working days of rain fed crop production.

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