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New Approach for Position Control of Induction Motor

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***Abstract* - Induction Motor (IM) has such characteristics as multivariable, high coupled and high nonlinearity, which implies that it is very difficult to control. In this paper, to achieve accurate control performance of position control of IM, a newly designed optimal control method is presented. The new proposed controller is designed via combining sliding mode control (SMC) and linear quadratic regulator (LQR). This is new technique fully matches the merits of the easy design of the LQR and the strong robustness of the SMC. To validate the performances of the new proposed control strategy, we provided a series of simulations and a comparative study between the performances of the new proposed controller strategy and those of the LQR and PI controller techniques. Simulation results show that the proposed approach gives a better position response and is robust to parameter variations.**

***Index Terms*- Optimal control, Linear quadratic regulator, Sliding mode control, Induction motor, Position control.**

I. INTRODUCTION

In many fields of industry, Induction motors are gradually replacing DC motors due to advantages such as simplicity, ruggedness, high reliability, and lower cost. Moreover, in contrast to DC motors, induction motors can be used for a long time without maintenance because of their brushless structure. Nevertheless, the ease of control of the DC motor cannot be matched because the IM has a nonlinear and interacting multivariable control structure. Control of the DC motors is straightforward due to the decoupled field and armature axes. The control of induction motors has attracted many attentions in the last decades. One of the significant developments in this area has been the field oriented control (FOC) or vector control (VC) [1]. With FOC techniques [2-4], the decoupling of torque and flux control commands of the IM is guaranteed, and the IM can be controlled linearly as a separated excited DC motor. However, the performance is sensitive to the variations of machine parameters, especially the rotor time constant, which varies with the temperature and the saturation of the magnetizing inductance. Recently, much attention has been given to the possibility of identifying the changes in motor parameters of an IM while the drive is in normal operation. Some researchers have proposed various IM drives with rotor resistance or rotor time constant identification to produce better control performance. However, the control performance of the IM is still influenced by the uncertainties, such as external disturbance, mechanical parameter variations, and unmodeled dynamics.

The LQR method is a powerful technique for designing controllers for complex systems that have stringent performance requirements. The LQR method seeks to find the optimal controller that minimizes a given cost function. This cost function is parameterized by two matrices, Q and R, that weight the state vector and the system input respectively. Considering the LQR method is an easy way to decide the demand control law to satisfy the requirements. It is based on the state-space model. To find the control law, a relative Riccati equation is first solved, and an optimal feedback gain, which will lead to optimal results evaluating from the defined cost function (performance index), is obtained. The motor system usually can be modelled as a second-order state-space system in which the mechanical speed and position is used as the system states. Thus, this method, seem quite suitable to the motor drive system. There are, however, few real motor systems adopting this method as the controller design. The main problem is that, while the desired performance can be achieved in the nominal system, it is difficult to incorporate robustness consideration into the design procedure. Besides the facts, once the external disturbance and/or the parameters uncertainty exist, then the desired responses may not be obtained [5]. In the past decade, the SMC strategies have been the focus of many studies and much research [6-9]. In general, the design of SMC involves the determination of a sliding surface that represents the desired stable dynamics, the description of a control law that guarantees the reaching condition and sliding condition. It is known that the SMC can offer such properties as insensitivity to parameters variations, external disturbance rejection, and fast dynamic response [10]. Nevertheless, this type of control has a disadvantage, which is the chattering phenomenon. The chattering phenomenon, that is, the high frequency finite amplitude oscillations with finite frequency caused by system imperfections, is produced due to the discontinuous of the signum function on the sliding surface. Several solutions have been proposed in the research literature to reduce the chattering. The most common method for solving the chattering problem is to introduce a boundary layer around the sliding surface and to use continuous control inside the boundary layer.

In this paper, to achieve accurate control performance of position control of IM, a newly designed control optimal method is presented. The proposed controller is designed via combining SMC and LQR. The merits of this control strategy are not only the optimal performance could be obtained but the robustness is guaranteed also. LQR method is used to decide the demand feedback gain to shape the dynamics and to meet the requirement of the performance index. At the same time, a SMC is used to conserve the robustness in the optimal control scheme. The remainder of the paper is organized as follows. In section II, the basic concept of LQR is briefly reviewed. Section III, discussed the weighting matrices Q and R. section IV, the IM model is briefly reviewed for the purpose of position control. The simulation results are stated in section V. Finally, section VI states the main conclusions.

II. Basic Concept Of LQR

The LQR is a powerful technique for designing controllers for complex systems that have stringent performance requirements. The standard theory of the optimal control is presented in [11-12]. Under the assumption that all state variables are available for feedback, the LQR design method starts with a defined set of states which are to be controlled. In general, the system model can be written in state space equation as follows:

$$\dot{x} = Ax + Bu \tag{1}$$

Where $x \in R^n$ and $u \in R^m$ denote the state variable, and control input vector, respectively. A is the state matrix of order n×n; B is the control matrix of order n×m. Also, the pair (A, B) is assumed to be such that the system is controllable. The LQR design is a method of reducing the performance index to a minimize value. The minimization of it is just the means to the end of achieving acceptable performance of the system. For the design of a LQR, the performance index (J) is given by:

$$J = \int_0^\infty \left(x^T Qx + u^T Ru \right) dt \tag{2}$$

Where Q is symmetric positive semi-definite (≥ 0) state weighting matrix of order n×n, and R is symmetric positive definite (> 0) control weighting matrix of order m×m [13]. The choice of the element Q and R allows the relative weighting of individual state variables and individual control inputs as well as relative weighting state vector and control vector against each other. The term in the brackets in equation (2) above are called quadratic forms and are quite common in matrix algebra. Also, the performance index will always be a scalar quantity, whatever the size of Q and R matrices. The conventional LQR problem is to find the optimal control input law u^* that minimizes the performance index under the constraints of Q and R matrices. The closed loop optimal control law is defined as:

$$u^* = -Kx \tag{3}$$

Where K is the optimal feedback gain matrix, and determines the proper placement of closed loop poles to minimize the performance index in (2). The matrix K depends on the matrices A, B, Q, and R. There are two main equations which have to be calculated to achieve K. Where P is a symmetric and positive definite matrix obtained by solution of the Algebraic Riccati Equation (ARE) is defined as:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{4}$$

Then the feedback gain matrix K is given by:

$$K = R^{-1}B^T P \tag{5}$$

Substituting (3) into (1), we obtain

$$\dot{x} = Ax - BKx = (A - BK)x \tag{6}$$

If the eigenvalues of the matrix (A-BK) have negative real parts, such a positive definite solution P always exits.

III. Weighting Matrices Q And R Determination

The weighting matrices Q and R are important components of an LQR optimization process. The compositions of Q and R elements have great influences of system performance. The designer is free to choose the matrices Q and R, but the selection of matrices Q and R is normally based on an iterative procedure using experience and physical understanding of the problems involved. Commonly, a trial and error method has been used to construct the matrices Q and R elements. This method is very simple and very familiar in LQR application. However, it takes long time to choose the best values for matrices Q and R. The number of matrices Q and R elements are dependent on the number of state variable (n) and the number of input variable (m), respectively. The diagonal-off elements of these matrices are zero for simplicity. If diagonal matrices are selected, the quadratic performance index is simply a weighted integral of the squared error of the states and inputs [14].

IV. Position Control Of IM

In this section, we show the designed procedure for the position control of IM system which is under the control of combining the SMC and LQR. The position control goal is to force the rotor position θ_r to track the desired rotor position reference θ_d. For the position control system, the mechanical equation of an IM drive can be represented as:

$$\ddot{\theta}_r = \frac{1}{J}\left(-B\dot{\theta}_r - T_L + \frac{3PL_m^2}{2L_r} i_{dse}^* i_{qse}^*\right) \tag{7}$$

Where J is the moment of inertia, B is the damping coefficient, T_L is the torque of external load disturbance, L_m is the magnetizing inductance per phase, L_r is the rotor inductance per phase referred to stator, P is the number of pole pairs, i_{qse}^* and i_{dse}^* denote the torque and flux current commands. For position controlled by the LQR, the system must first be expressed in the state space model. The corresponding IM dynamic equation in the state space model without considering the disturbance T_L is obtained as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & a \end{bmatrix} x + \begin{bmatrix} 0 \\ b \end{bmatrix} u = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-B}{J} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{K_t}{J} \end{bmatrix} i_{qse}^* \tag{8}$$

where $x_1=\theta_r-\theta_d$ and $x_2 = \dot{x}_1$. The objective of LQR, it is to find an optimal control u^* minimizing the performance index. To yield a minimum index of (2), the control law u^* is defined as follows:

$$u^{*} = -Kx = -(R^{-1}B^{T}P)x \tag{9}$$

When system (8) is under the control of (9), the resultant closed loop dynamics are given by:

$$\dot{x} = (A - BK)x = A_{c}x \tag{10}$$

If the parameters of the systems are deviated from the nominal values and/or external load disturbance is added into the system, then the desired responses may not be obtained. The nominal system in (8) is rewritten as:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & a+\Delta a\end{bmatrix}x + \begin{bmatrix}0\\ b+\Delta b\end{bmatrix}u + \begin{bmatrix}0\\ d\end{bmatrix} \tag{11}$$

Where Δa, Δb denote the uncertainties in introduced by system parameters J, B and K_t. d= T_L/J represents the external disturbance. Equation (11) can be expressed in the form of:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & a\end{bmatrix}x + \begin{bmatrix}0\\ b\end{bmatrix}u + \begin{bmatrix}0\\ \Delta a x_2 + \Delta b u + d\end{bmatrix} \tag{12}$$

where $w = \Delta a x_2 + \Delta b u + d$ is the total perturbation. To overcome the drawbacks of the LQR method mentioned above, the SMC will be added to the conventional LQR. Then, the total control is given as:

$$u = u^{*} - \beta sign(s) = -k_1 x_1 - k_2 x_2 - \beta sign(s) \tag{13}$$

where β is a positive scalar. Here the sliding surface (s) with integral operation for the sliding mode position controller is designed as:

$$s = Cx - C(A-BK)\int_0^t x\,dt = Cx - CA_c\int_0^t x\,dt \tag{14}$$

The additional integral provides one more degree of freedom in design than traditional linear sliding surface, also to improve performance in steady state. $C = [c_1 \; c_2] \in R^{1\times 2}$ the coefficient vector of the linear part of the sliding mode and is chosen in a way such that $CB \neq 0$, and K is a linear feedback matrix. To calculate sliding surface, let $C = [0 \;\; \frac{1}{b}]$.

$$s = k_1\int_0^t x_1 dt + \frac{1}{b}x_2 - (\frac{a}{b} - k_2)x_1 \tag{15}$$

Using the novel control law (13), the sliding surface s = 0 is attractive in a finite time and invariant for the closed loop system.

Proof:

By choosing the Lyapunov function candidate:

$$V = 0.5s^{2} \tag{16}$$

The first time derivative of the positive definite Lyapunov function in (16) is:

$$\dot{V} = s\dot{s} \tag{17}$$

Differentiating (20) and substituting into (22):

$$\dot{V} = s[k_1 x_1 + \frac{1}{b}\dot{x}_2 + (k_2 - \frac{a}{b})x_2] \tag{18}$$

Substituting $\dot{x}_2$ from (8) into (18), then:

$$\dot{V} = s[k_1 x_1 + \frac{1}{b}(ax_2 + bu) + (k_2 - \frac{a}{b})x_2] \tag{19}$$

Substituting the total control (u) from (13) into (19), then:

$$\dot{V} = s\{k_1 x_1 + \frac{1}{b}[ax_2 + b\left(-k_1 x_1 - k_2 x_2 - \beta sign(s)\right)] + (k_2 - \frac{a}{b})x_2\}$$

$$\dot{V} = s[k_1 x_1 + \frac{a}{b}x_2 - k_1 x_1 - k_2 x_2 - \beta sign(s) + k_2 x_2 - \frac{a}{b}x_2]$$

$$\dot{V} = s[-\beta sign(s)] = -\beta|s| < 0$$

The constant β should be positive in order to satisfy the sliding condition $\dot{V} = s\dot{s} < 0$. If total or partial of perturbation w exists, (19) will be modified as:

$$\dot{V} = s\{k_1 x_1 + \frac{1}{b}[ax_2 + b\left(-k_1 x_1 - k_2 x_2 - \beta sign(s)\right) + w] + (k_2 - \frac{a}{b})x_2\}$$

$$\dot{V} = s[k_1 x_1 + \frac{a}{b}x_2 - k_1 x_1 - k_2 x_2 - \beta sign(s) + \frac{w}{b} + k_2 x_2 - \frac{a}{b}x_2]$$

$$\dot{V} = s[\frac{w}{b} - \beta sign(s)] = \frac{w}{b}|s| - \beta|s| = \left[\frac{w}{b} - \beta\right]|s| < 0$$

The sliding is guaranteed if $\beta > \left|\frac{w}{b}\right|$.

In practice, a discontinuous control component as signum function is undesirable because it may cause a chattering problem. To reduce the chattering of the SMC, the signum function replaced by smooth function. A new control law is given by:

$$u_T = -k_1 x_1 - k_2 x_2 - \beta\frac{s}{|s| + \delta} \tag{20}$$

Where δ is a small positive design constant, and the switching function (s) has the same definition as (15).

V. Simulation Results

To validate the performances of the new proposed control technique, we provided a series of simulation tests and a comparative study between the performances of the new proposed controller strategy, LQR and PI controller under three different test conditions. The different controller algorithms are compared using the same rotor position reference command. PI controller gains are selected as follows k_P=1200, k_I=1. To determine the feedback gain K, the elements of matrices Q and R are chosen as: Q = [300 0;0 1] and R = 1. By using lqr command in Matlab, the control feedback gain K can be obtained as $K = [54.7723 \quad 6.6826]$.

A. Nominal Condition

In this section the tracking performances of the new proposed controller, LQR and PI controller schemes are compared under nominal condition. Figs 1-3 show the rotor position tracking, rotor position tracking error, and control effort using the three controllers, respectively. The results show that high precision rotor position tracking can be achieved using the three controller laws, particularly the new controller. The new proposed controller technique shows smaller peak error of about ±0.0045rad compared to the LQR and PI controller laws of about ±0.0077rad or larger. Fig. 3 shows plot of control effort i^*_{qse} versus time for the three controllers. In the nominal condition parameters case, the control efforts of the new proposed and LQR controllers are similar.

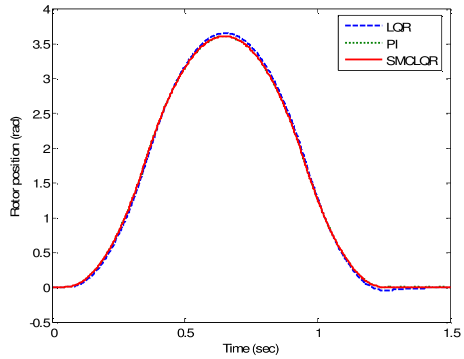

Fig. 1 Rotor position tracking performance

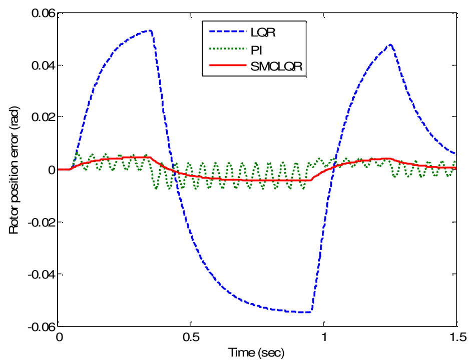

Fig. 2 Rotor position tracking error

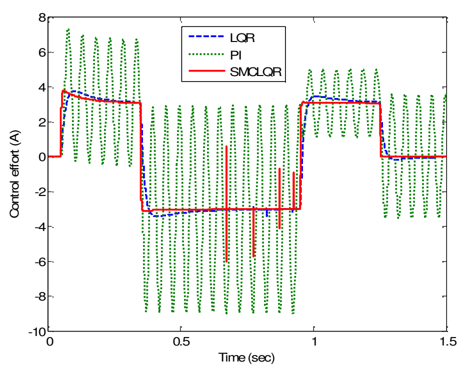

Fig. 3 Control effort

B. Increase the Moment of Inertia

The simulation test conditions are the same as in the previous section, but here the moment of inertia of the machine is doubled. Figs 4-6 show the simulation results under double inertial condition for the three controllers under consideration. The results show that high precision rotor position tracking still can be achieved using the new proposed controller technique despite the high moment of inertia. However, three controller's exhibit increased tracking rotor position errors for the high inertia mismatch. The maximum rotor position errors for the new proposed, LQR and PI controllers are recorded as ±0.0144rad, ±0.11rad, and ±0.07rad, respectively. Although, the rotor position error produced by the new controller is still smaller than other two controller techniques. In the moment of inertia case, a higher control effort is demanded to counter this change.

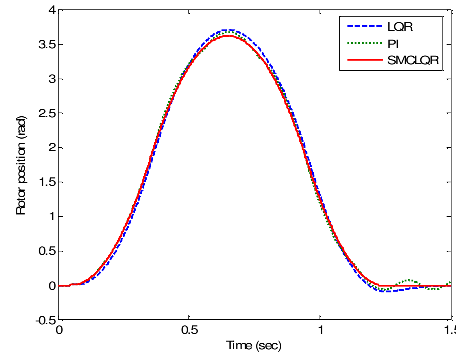

Fig. 4 Rotor position tracking performance

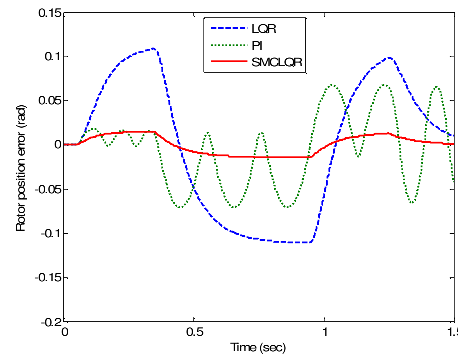

Fig. 5 Rotor position tracking error

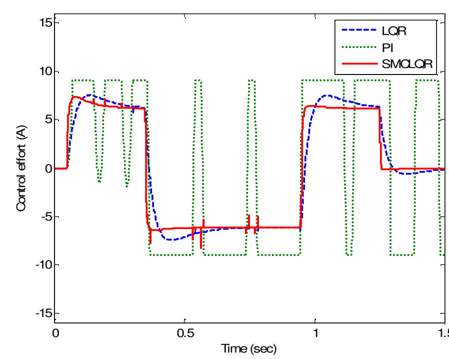

Fig. 6 Control effort

C. Increase the Rotor Resistance

In order to test the robustness of the controller algorithms with rotor resistance mismatch, the rotor resistance is increased to 1.75Ω during the simulation tests. Figs 7-9 show the simulation results for the three controllers. Despite the rotor resistance mismatch in the system, the results show that high precision rotor position tracking still can be achieved using the new proposed controller technique. However, the simulation results show larger peak position errors when the rotor resistance is increased. Thus, when the rotor resistance changes, the torque response changes and effects the rotor position. The rotor position error range remains within ±0.0144rad from the new proposed controller but within ±0.11rad and ±0.07rad for the LQR and PI controller algorithms respectively. The new proposed controller is again proven to be more robust, with smaller peak rotor position error. In the rotor resistance mismatch case, a higher control effort is demanded to counter this change. It varies up to about ±9A (maximum) during this case.

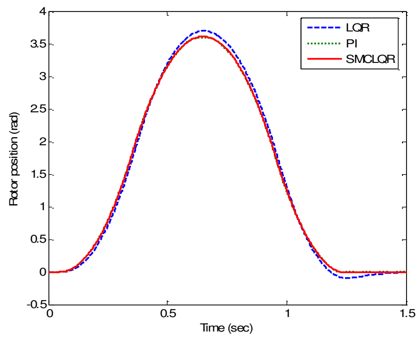

Fig. 7 Rotor position tracking performance

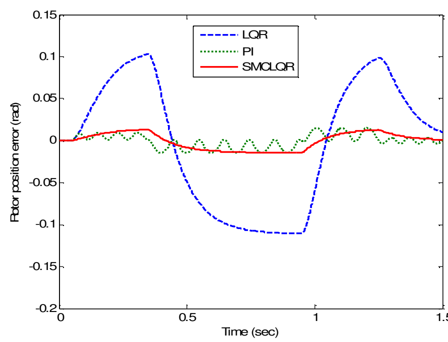

Fig. 8 Rotor position tracking error

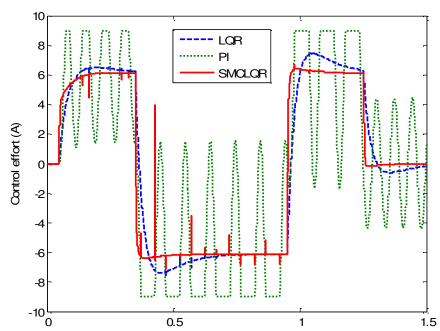

Fig. 9 Control effort

VI. CONCLUSIONS

In this paper, a new methodology that combines LQR with SMC is submitted to design the rotor positioning control of IM. To validate the performances of the new controller strategy, we provided a series of simulations and a comparative study between the performances of the new controller and those of the LQR and PI controllers. It can be seen that in case one all controllers have good tracking performance, particularly the new controller with smaller rotor position error. In case two, the results show that high precision rotor position tracking still can be achieved using the new controller technique, but the system performance with LQR and PI controllers degraded. In the case three, the new controller is superior to the LQR and PI controllers in the robustness to rotor resistance variations.

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