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A New Stream-Tube Model for Waterflooding Performance in 5-Spot Patterns

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Abstract

A new mathematical model is developed for waterflooding performance calculation in 5-spot pattern reservoirs using stream tubes. The method is based on estimating the location of the displacement front in any stream tubes relative to the location in any other tube at the same real time. Areal sweep efficiency, fractional oil recovery, fractional water flow and the injectivity ratio are calculated as functions of thetotal injected P.V. of the displacing fluid. The effect of mobility ratio on the performance is investigated. Comparison is also made between the performance obtained assuming piston-like displacement and that assuming Buckley-Leverett frontal advance theory.

Introduction

Much interest had been shown lately in using stream tube models for performance prediction in miscible and immiscible displacement[1-4]. The appeal of these methods is based on the fact that the obtained solutions are free of the numerical dispersion that is inherent in finite difference solutions. Stream tubes (or channels) are generated by solving the Laplace equation in two dimensions for the pressure and tracking the movement of particles in the pressure field. Analytical solutions are also available in term of eliptic sine and cosine functions[5]. Also, the method of superposition can be used to generate the potential field in different regular flood patterns or in an area with a specified number of wells with known locations and rates[6]. Two assumptions are usually made in generating stream tubes: unit mobility ratio (equal mobilities of displacing and displaced fluids) and stream tubes not changing with time. Performance calculations of water-flooding by the stream tube method is attributed to Higgins and Leighton[7-10] who published a series of articles illustrating the application of the method. Although Muskat[11] presented calculations of sweep efficiency for different flood patterns, his calculations are based on unit mobility ratio. Other works considered the effect of mobility ratio but assumed the displacement to be piston-like[12,13]. Higgins and Leighton applied the Buckley-Levertt frontal advance theory to the displacement in the stream tubes. In this case, a zone with varying saturation and mobility is developed behind the displacement front. Martin and Wegner[14] presented a numerical procedure that generates stream tubes by finite difference solution and solve the displacement problem (saturation and pressure) inside the tubes by the frontal advance theory. In their model, the stream tubes change with time as the flood progresses.

Predicting the waterflooding performance by stream tube method is usually conducted on single tube bases. Calculations are performed on each individual tube separately and then results from all the tubes are added together to obtain the overall performance of the pattern. In terms of dimensionless variables, the performance of all the tubes is the same. The dimensionless time of breakthrough, the P.V. of oil produced and fractional flow of water versus P.V. of injected fluid is the same for all tubes. In this case the dimensionless times and pore volumes are based on the total pore volume of the correspsonding tube rather than on the total pore volume of the flood pattern. The performance of each tube is to be transformed into real parameters i.e. volume vs. time so that it can be added to that of other tubes.

In this work a mathematical model is presented to relate the location of the flood frons in the different channels at the same real time. A procedure similar to that used by Dykstra-Parsons[15] for immiscible displacement in non-communicating stratified reservoirs. Stream tubes are considered as noncommunicating layers since they are normal to the potential lines. Rather than the linear layers with different absolute permeabilities in the Dykstra-Parsons model, the stream tubes have the same absolute permeability but with different geometrical shape characteristics. Both piston-like displacement and Buckley Leverett frontal advance displacement will be considered.

Mathematical Model

Immiscible displacement in the Tube

The continuity equations for oil and water flow inside the tube, assuming incompressible flow, are

$$\phi A \frac{\partial S_w}{\partial t} = -\frac{\partial}{\partial Z}(AV_w) \quad (1)$$

$$\phi A \frac{\partial S_o}{\partial t} = -\frac{\partial}{\partial Z}(AV_o) \quad (2)$$

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Adding Eq. (1) and (2) and noting that

$$S_o + S_w = 1 \quad (3)$$

wt get

$$\frac{\partial}{\partial Z}(q_w + q_o) = \frac{\partial}{\partial Z}(q_t) = 0 \quad (4)$$

. .

substituting in Eq. (1) for

$$q_w = f_w \; q_t \quad (5)$$

we get

$$\phi A \frac{\partial s_w}{\partial t} = -q_t \frac{df_w}{ds_w}\frac{\partial s_w}{\partial Z} \quad (6)$$

In dimensionless form

$$\frac{\partial s_w}{\partial \tau} = -\frac{df_w}{ds_w}\frac{\partial s_w}{\partial V_{pD}} \quad (7)$$

where

$$\tau = \frac{\int_0^t q\,dt}{V_{pt}} \quad (8)$$

$$V_{pD} = \frac{\int_o^z \phi A dz}{V_{p_{tot}}} = \frac{\int_o^z \phi A dz}{\int_o^{z_t} \phi A dz} \quad (9)$$

Applying the method of characteristics (M.O.C) to Eq. (7) gives

$$\left(\frac{dV_{pD}}{d\tau}\right)_{s_w} = \frac{d\,f_w}{d\,s_w} \quad (10)$$

Since most experimental relative permeability data yield an s-shaped $f_w - S_w$ curve, a multi-valued saturation distribution would occur. Using the concept of saturation discontinuity (shock) at the front, the saturation at the front S_w^* is determined from the material balance consideration and is given by the equation

$$\frac{d f_w^*}{d S_w} = \frac{f_w^*}{S_w^* - S_{wi}} \quad (11)$$

The value of S_w^* is determined by plotting a tangent to the f_w - S_w curve from the point representing the irreducible water saturation (S_{wi}, 0). The pore volume flooded by water at any time τ is given by

$$V_{pDf} = \tau\left(\frac{d\,f_w^*}{d\,S_w}\right) \quad (12)$$

and the saturation distribution in the tube at any point V_p is given by the relation

$$f'_w(V_{pD}) = \frac{V_{pD}}{V_{pDf}}\; f'^{*}_w \quad (13)$$

knowing $f'_w\,(V_{pD})$, the f_w-S_w relation can be solved numerically (by iteration) for $S_w(V_p)$. The oil and water relative permeabilities and hence the mobilities can then be determined.

Pressure drop and flow rate in stream-tubes

From Darcy's law

$$q_w = w\,h\,k\,\lambda_w\,\left(-\frac{dp}{dz}\right) \quad (14)$$

$$q_o = w\,h\,k\,\lambda_o\,\left(-\frac{dp}{dz}\right) \quad (15)$$

Adding the two equations

$$q_t = w\,h\,k\,\lambda_t\,\left(-\frac{dp}{dz}\right) \quad (16)$$

so

$$-dp = \frac{q_t}{k\,h}\frac{d\,z}{w\,\lambda_t} \quad (17)$$

The total pressure drop ΔP_t is obtained by integrating over the total length of the tube

$$\Delta P_+ = \frac{q_t}{k\,h}\int_o^{z_t} \frac{d\,z}{w\,\lambda_t}$$

$$= \frac{q_t}{k\,h}\int_o^{v_{p_t}} \frac{d\,V_p}{w^2\,\lambda_t} \tag{18}$$

Introducing dimensionless variables V_{pD} and W_D where

$$W_D = \frac{W}{\overline{W}} \tag{19}$$

$$\overline{W} = \frac{\int_o^{z_t} W dz}{z_t} = \frac{V_{p_t}}{z_t} \tag{20}$$

we get

$$\Delta P_{tot} = \frac{q_t\,V_{p_t}}{\overline{w}^2\,k\,h}\int_o^1 \frac{d\,V_{pD}}{w_D^2\,\lambda_t} \tag{21}$$

when the flood front in the tube reaches a dimensionless pore volume V_{pD} , the integral of Eq. (21) is divided into two parts. The mobility in the first part depends on the saturation distribution. The mobility in the uninvaded part of the stream tube, λ_o^o, is that of oil at irreducible water saturation.

$$\lambda_o^o = \frac{K_{ro}^o}{\mu_o} \tag{22}$$

Hence

$$\Delta P_t = \frac{q_t\,V_{p_t}}{\overline{w}^2\,k\,h}\left[\int_o^{V_{PDf}} \frac{d\,V_{pD}}{w_D^2\,\lambda_t} + \frac{1}{\lambda_o^o}\int_{V_{PDf}}^1 \frac{d\,V_{pD}}{w_D^2}\right]$$

$$= \frac{q_t\,V_{p_t}}{\overline{w}^2\,k\,h\,\lambda_o^o}\left[\int_o^1 \frac{d\,V_{pD}}{w_D^2} - \int_o^{V_{PDf}} (1-\frac{\lambda_o^o}{\lambda_t})\frac{d\,V_{pD}}{w_D^2}\right] \tag{23}$$

Let

$$\int_o^{V_{pD}} \frac{d\,V_{pD}}{w_D^2} = g \tag{24}$$

$$\int_o^1 \frac{d\,V_{pD}}{w_D^2} = g_t \tag{25}$$

$$g_D = \frac{g}{g_t} \tag{26}$$

Eq. (23) can be written as

$$\Delta P_t = \frac{q_t\,V_{p_t}\,g_t}{k\,h\,\overline{w}^2\,\lambda_o^o}\left[1 - g_{Df} + \frac{\lambda_o^o}{g_t}\int_o^{V_{pDf}} \frac{d\,V_{pD}}{\lambda_t\,w_D^2}\right] \tag{27}$$

If we define an average total mobility $\overline{\lambda}_t$ as

$$\int_o^{V_{pDf}} \frac{d\,V_{pD}}{\lambda_t w_D^2} = \frac{1}{\overline{\lambda}_t}\int_o^{V_{pDf}} \frac{d\,V_{pD}}{w_D^2} \tag{28}$$

then

$$\frac{1}{\overline{\lambda}_t} = \frac{1}{g_t\,g_{Df}}\int_o^{V_{pDf}} \frac{d\,V_{pD}}{\lambda_t w_D^2} \tag{29}$$

Substituting in Eq. (27) and rearranging for q_{tot}

$$q_t = \frac{k\,h\,\lambda_o^o\,\overline{w}^2\,\Delta P_t}{V_{p_t}\,g_t\left[1-(1-\frac{\lambda_o^o}{\overline{\lambda}_t})\,g_{Df}\right]} \tag{30}$$

At water breakthrough in the tube, $g_{Df} = 1$ and

$$q_t = \frac{k\,h\,\overline{w}^2\Delta P_t\,\overline{\lambda}_t}{Vp_t g_t} \tag{31}$$

and

$$\frac{1}{\overline{\lambda}_t} = \frac{1}{g_t}\int_o^1 \frac{d\,V_{pD}}{\lambda_t w_D^2} \tag{32}$$

Relative Front locatoins.
From Eq. (12)

$$d\,V_{pDf} = f_w^{'*}\,d\tau = \frac{f_w^{'*}\,q_t\,dt}{V_{pt}} \tag{33}$$

so

$$\frac{d\,V_{pDf}}{dt} = \frac{q_t\,f_w^{'*}}{V_{pt}} \tag{34}$$

Substituting for q_r from Eq. (30)

$$\frac{d\,V_{pDf}}{dt} = \frac{k\,h\,\lambda_o^o\;f_w^{'*}\Delta P_t}{Z_t^2\;g_t\left[1-(1-\frac{\lambda_o^o}{\overline{\lambda}_t})\,g_{Df}\right]} \tag{35}$$

For any two tubes i and j

$$\frac{d\,V_{pDf_i}}{d\,V_{pDf_j}} = \frac{(Z_t^2\;g_t)_j}{(Z_t^2\;g_t)_i}\frac{\left[1-(1-\frac{\lambda_o^o}{\overline{\lambda}_t})\,g_{Df}\right]_j}{\left[1-(1-\frac{\lambda_o^o}{\overline{\lambda}_t})\,g_{Df}\right]_i} \tag{36}$$

or

$$\frac{d\,V_{pDf_i}}{d\,V_{pDf_j}} = \frac{(Z_t^2\,g_t)_j}{(Z_t^2\,g_t)_i}\,\frac{\phi_j}{\phi_i} \qquad (37)$$

$$\Delta V_{pDf_i} = \frac{(Z_t^2\,g_t)_j}{(Z_t^2\,g_t)_i}\,\frac{\phi_j}{\phi_i}\,\Delta V_{pDf_j} \qquad (38)$$

where

$$\phi = 1 - (1 - \frac{\lambda_o^o}{\bar{\lambda}_t})\,g_{Df} \qquad (39)$$

So, if the front in tube j is assumed to moves a specified interval ΔV_{pDfj}, the total volume flooded in tube j, V_{pDj} is known and the saturation distribution, the average mobility, and hence ϕ_j can be calculated. It is then required to determine the incremental movement ΔV_{pDfi} in tube i such that Eq. (38) is satisfied. A trial and error (iterative) procedure is used.

Areal Sweep Efficiency and Oil Recovery

The areas invaded by water in the different tubes are added and divided by the total area of the flood pattern to obtain the areal sweep efficiency E_s. Before water B.T. in the producing well

$$E_s = \frac{\sum_{i=1}^{Nc} V_{PD\,f_i}}{\sum_{i=1}^{Nc} V_{Pt_i}} \qquad (40)$$

where N_c is the number of channels(stream tubes) used.
After water B.T. in channel j, channels 1,2,…j are completely flooded and

$$E_s = \frac{\sum_{i=1}^{j} V_{Pt_i} + \sum_{j+1}^{Nc} V_{PD\,f_i} V_{Pt_i}}{\sum_{i=1}^{Nc} V_{Pt_i}} \qquad (40\text{-A})$$

when all channels are swept, the sweep efficiency is unity.

To calculate the oil recovery, the areas swept before breakthrough are multiplied by the quantity ($\bar{S}_w - S_w i$) which is equal to $\tau_{B.T.}$. So before B.T. in any of the tubes

$$R = Es\ \ \tau_{B.T.} \qquad (41)$$

where

$$\tau_{B.T.} = \frac{1}{f_w^{\,'*}} \qquad (42)$$

After water breakthrough in stream tube j, the average saturation in that tube is given by

$$\bar{S}_w = S_{wL} + \tau(1 - f_{wL}) \qquad (43)$$

where S_{wL} is the water saturation at the outlet (last cell) of the channel, f_{wL} is the fractional flow of water corresponding to S_{wL} and τ is the dimensionless time of the specified channel. The saturation at the outlet of the channel is determined from the relation

$$\left(\frac{d\,f_w}{d\,s_w}\right)_L = \frac{1}{\tau} \qquad (44)$$

Dimensionless Time

For each channel before water B.T.

$$\tau = \frac{V_{pDf}}{f_w^{\,'*}} = V_{pDf}\ \ \tau_{B.T}. \qquad (45)$$

and the total dimensionless time is

$$\tau_t = \frac{\sum_{i=1}^{Nc} \tau_i\,V_{pt_i}}{\sum_{i=1}^{Nc} V_{pt_i}} \qquad (46)$$

From Eq. (35) for tube i

$$\Delta V_{pDf_i} = \frac{\Delta t_r}{(Z_t^2\ g_t)_i\,\phi_i} \qquad (47)$$

$$\Delta t_r = k\,h\,\lambda_o^o\,f_w^{\,'*}\Delta P_t\,\Delta t \qquad (48)$$

Δt_r is a relative time and is the same for all tubes at the same time.

For tube j after B.T., from Eq. (31)

$$q_{t\,j} = \frac{V_{pt_j}\,d\,\tau_j}{dt} = \frac{V_{ptj}\,k\,h\,\lambda_o^o\,\Delta P_{tot}}{\left(Z_t^2\ g_t\right)_j}\,\frac{\bar{\lambda}_t}{\lambda_o^o} \qquad (49)$$

$$\frac{\bar{\lambda}_o}{\bar{\lambda}_t}\,d\tau_j = \frac{k\,h\,\lambda_o^o\,\Delta P_t}{(Z_t^2\ g_t)_j}\,dt \qquad (50)$$

$$\Delta\,\tau_j = \frac{\Delta t_r\,\tau_{B.T}}{(Z_t^2\ g_t)_j}\,\frac{\bar{\lambda}_{t\,j}}{\lambda_o^o} \qquad (51)$$

$\Delta\tau_j$ is calculated by trial and error to satisfy Eq. (51) with Δt_r calculated from channel i which is not swept yet. The dimensionless time in the tube, τ_j, is updated by adding $\Delta\tau_j$ to the previous value of τ_j.

Factional Flow

For tubes totally swept, f_w is calculated from the water saturation S_{wL} at the outlet of the tube. If j tubes are swept then

$$f_w = \frac{\sum_{i=1}^{j} q_i \; f_{wi}}{\sum_{i=1}^{j} q_i + \sum_{j+1}^{Nc} q_i}$$

$$= \frac{\sum_{i=1}^{j} \left(\frac{V_{pt}}{Z_t^2 \; g_t} \right)_i \overline{\lambda}_{t_i} \; f_{wi}}{\sum_{i=1}^{j} \left(\frac{V_{pt}}{Z_t^2 \; g_t} \right)_i \overline{\lambda}_{t_i} + \sum_{j+1}^{Nc} \left(\frac{V_{pt}}{Z_t^2 \; g_t} \right)_i \frac{1}{[1-(1-\frac{\lambda_o^o}{\overline{\lambda}_t}) \; g_{Di}]}} \qquad (52)$$

Injectivity Ratio

The total injection rate when j tubes are swept is obtained from Eqs. (30) and (31)

$$Q_{tot} = kh\lambda_o^o \Delta P_{tot} \{ \sum_{i=1}^{j} \left(\frac{V_{pt}}{Z_t^2 \; g_t} \right)_i . \frac{\overline{\lambda}_{t_i}}{\lambda_o^o} + \sum_{j+1}^{Nc} \left(\frac{V_{pt}}{Z_t^2 \; g_t} \right)_i \frac{V_{pt_i}}{[1-(1-\frac{\lambda_o^o}{\overline{\lambda}_t}) g_d]_i} \} \qquad (53)$$

Initially

$$Q_{tot} = kh\lambda_o^o \Delta P_{tot} \sum_{i=1}^{N_t} \left(\frac{V_{pt}}{(Z_t^2 \; g_t).} \right)_i \qquad (54)$$

The injectivity ratio is given by

$$I_r = \frac{(\frac{Q_t}{\Delta P_t})}{(\frac{Q_t}{\Delta P_t})_i}$$

$$= \frac{\sum_{i=1}^{j} \left(\frac{V_{pt}}{Z_t^2 \; g_t} \right)_i \frac{\overline{\lambda}_{t_i}}{\lambda_o^o} + \sum_{j+1}^{Nc} \left(\frac{V_{pt_i}}{Z_t^2 \; g_t} \right)_i \frac{1}{[1-(1-\frac{\lambda_o^o}{\overline{\lambda}_t}) g_D]_i}}{\sum_{i=1}^{Nc} \left(\frac{V_{pt}}{Z_t^2 \; g_t} \right)_i} \quad .... \quad (55)$$

Piston-Like Displacement

In piston-like displacement, the total mobility in the invaded cells is constant and equals λ_o^o. In this case

$$\overline{\lambda}_t = \lambda_t = \lambda_w^o \qquad (56)$$

and the term ϕ in the denemoniater of Eqs. (30), (35), (52), (53) and (55) become

$$\phi = 1-(1-\frac{\lambda_o^o}{\lambda_w{}^o}) g_D = 1-(\frac{M-1}{M}) g_D \qquad (57)$$

where M is the mobility ratio of the displacing to displaced fluid

$$M = \frac{\lambda_w^o}{\lambda_o^o} = \frac{K_{rw}^o}{K_{ro}^o} \frac{\mu_o}{\mu_w} \qquad (58)$$

For unit mobility ratio, Eq. (57) reduces to $\phi = 1$ and Eq. (37) becomes

$$\frac{V_{pDi}}{V_{pDj}} = \frac{(Z_t^2 \; g_t)_j}{(Z_t^2 \; g_t)_i} \qquad (59)$$

At time of B.T. in the first channel (V_{pD1}=1) the sweep efficiency is calculated to be 0.7584 as compared to the analytical value of 0.7178 obtained by Morel-Seytoux[5]. It is expected that if more stream tubes are used, the calculated values of E_s at B.T. would approach the theoretical value.

Solution Procedure

The data presented by Higgins and Leighton[10] are used here.The shape factors for four channels in half of the quadrant of the flood pattern are givegn. Each channel is divided into 40 cells of equal volumes. The shape factors (ΔZ/W) of all the cells are given. Noting that the volume of the cell (assuming unit height) is W. ΔZ, if follows that

$$\Delta Z = \sqrt{AG} \qquad (60)$$

and

$$W = \sqrt{A/G} \qquad (61)$$

The values of Z, V_p and g_p are calculated for all cells in each channel and then the dimensionless values V_{pD}, g_D, W_D are calculated. The geometrical characteristics of the four stream tubes are shown in figure 1. It is noticed that in dimensionless form, the geometrical characteristics of the four stream tubes are almost the same.

Performance calculations are started from the channel with the shortest length between injection and production wells. The front is advanced in channel 1 one cell at each time. The value of ΔV_{pD} in this channel is 0.025 (1/40). A value of ΔV_{pD} in each of the other channels is assumed and the total flooded volume is incremented by this value. Equation (13) is used to estimate f_w' in all cells and the water saturation and total mobility is calculated in all invaded cells in each channel. The average total mobility is estimated using Eq. (29) and the parameter ϕ is calculated from Eq. (39). Equation (38) is then used to estimate ΔV_{pD} in the other channels (except the first) and the values obtained are compared with the assumed values. Iteration is continued until convergence is achieved. This procedure is repeated for the 40 cells in channel 1. The calculations are then moved to the next channel starting from the last value of V_{pD} at the time of water breakthrough in the previous channel. The same procedure is followed for the rest of the (succeeding) unflooded channels. For the flooded channels, the dimensionless time for the channel is assumed and the saturation distribution and total mobility in all 40 cells are calculated. The average total mobility $\overline{\lambda}_t$ is then determined. Equation (51) is then checked for the incremental dimensionless time and the iteration is continued until convergence is achieved.

At each time step, Eq. (40) is used to estimate the sweep efficiency, Eq. (41) and (43) to estimate oil recovery, Eq. (52) to estimate the fractional flow f_w and Eq. (55) to estimate the injectivity ratio I_r. Calculations are continued until the last cell in the last channel is invaded by water. At that time, the sweep efficiency reaches a value of 1 but because of the use of Buckley-Leverett displacement rather than the piston-like displacement, f_w does not reach a value of 1 neither does the oil recovery reaches its maximum ultimate recovery value of $(1 - S_{wi} - S_{or})$.

Results and Discussion

The relative permeability relations used in this study are those given by Martin and Wegner[14].

$$K_{ro} = 0.75(1 - S_D)^2 \quad (63)$$

$$K_{rw} = 0.3\, S_D^3 \quad (62)$$

where

$$S_D = \frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}} \quad (64)$$

The relative permeability curves, the fractional flow curves and the total mobility curves for oil/water viscosity ratio ratio of 5 are given in Figure 2. Figure 3 shows the fractional flow curves and the total mobility curves for different values of the oil/water viscosity ratio. It is to be noted that the term mobility ratio as defined for piston-like displacement does not have the same physical meaning for the case of Bucley-Leverett displacement. In this case, the oil and water mobilities in the flooded cells are not constant but vary from the value of λ_w^o for water and $\lambda_o = 0$ for oil at the injection point to the values of λ_w^* and λ^*_o at the oil/water interface. The oil mobility of the unflooded zone is constant at λ_o^o while $\lambda_w = 0$.

Figure 4 shows the performance results for a run with oil/water viscosity ratio of 5 for both piston-like and Buckley-Leverett displacement. This corresponds to a mobility ratio of 2 for piston-like displacement. Earlier breakthrough will occur in the case of B-L displacement but the values of f_w reached after the breakthrough in the successive tubes are less than the corresponding values for a piston-like displacement process. This is due to the assumption of 100% water production from the flooded tubes in case of piston-like displacement while the tubes will continue to produce water and oil after water breakthrough in the case of B-L displacement. When the last stream tube is flooded, f_w reaches unity in the piston-like displacement but remains below unity in B-L displacement.

Oil recovery is reported in terms of the recoverable oil volume $(1 - S_{wi} - S_{or})$. In piston-like displacement, oil recovery and sweep efficiency are identical. This is not the case for B-L displacement where oil recovery is obtained by multiplying the sweep efficiency by the average saturation in the flooded zone. Before B.T. in the first stream tube, oil recovery for both B-L and piston like displacement is equal to the dimensionless time since all injected water remains in the reservoir and an equal amount of oil is produced. At the end of displacement when water B.T. occur in the last tube, the areal sweep efficiency reaches unity for both displacement cases. The fractional oil recovery in the B-L displacement case does not reach a value of one at that time.

The injectivity ratio starts at a value of one and increases with the progress of displacement. There is a continuous increase of the injectivity ratio for the piston-like displacement case and a value of 2 (the mobility ratio for this run) is reached at the end of displacement as predicted theoretically. As discussed earlier, the definition of mobility ratio for the piston-like displacement does not apply for the B-L displacement case. The injectivity ratio increases with time in this case but there is a sudden decrease when breakthrough occurs in the successive stream tubes and it does not reach a value of 2 at the time of water breakthrough in the last stream tube. At the injection point the total mobility λ_w^o is 0.3 (corresponding to a water viscosity of 1). At water breakthrough in the last cell in the stream tube, the total mobility corresponding to the breakthrough saturation ($S_D^* = 0.682$) is 0.1104. If an average value of 0.2052 is considered, this will result in a mobility ratio of 1.368. The value of the injectivity ratio reached at water B.T. in the last stream tube for this case is 1.3836. It is to be noted that the water saturation in the previously flooded channels, other than the last one, is higher than that corresponding to water breakthrough and the total mobility would be higher than 0.1104 as can be seen from Figs. 2 and 3.

Areal Sweep Efficiency
Figure 5 shows the areal sweep efficiency (flooded area) as function of the dimensionless time (injected P.V.) for different values of mobility ratio(as defined for piston-like displacement). Since the sweep efficiency is related to the injected P.V. before water breakthrough by the following relation.

$$E_s = \frac{\tau}{\overline{S}_D} \qquad (65)$$

where $\overline{S}_D$ is the average dimensionless saturation behind the displacement front, a straight line relationship exists between areal sweep efficiency E_s and dimensionless time τ until the time of B.T. in the first stream tube. Since S_D^* decreases with increasing mobility ratio, the slope of the straight lines increases with increasing mobility ratio or increasing oil/water viscosity ratio. After B.T. in the first channel, straight line segments with decreasing slopes are apparent from Figure 5. This can be explained by the increasing value of the average water saturation behind the displacement front after water B.T. The length of the first straight line segment increases with decreasing mobility ratio and ends at the value of areal sweep efficiency at breakthrough which is known to increase as the mobility ratio decreases.

Oil Recovery
Figure 6 shows the fractional oil recovery as function of the dimensionless time. Because the dimensionless water saturation S_D is used in estimating oil recovery and dimenssionless time, both terms are defined as fraction of the ultimate recoverable oil volume($1 - S_{wi} - S_{or}$). Until the time of B.T. in the first tube, a straight line with unit slope exists. The length of this straight line increases with decreasing mobility ratio or oil/water viscosity ratio. Straight line segments with decreasing slopes develop during displacement in the remaining stream tubes.

Figure 7 shows the relation between oil recovery and areal sweep efficiency for different values of mobility ratio. Before B.T. in the first channel, the oil recovery is obtained by multiplying the areal sweep efficiency by the average dimensionless saturation behind the front which is given by

$$\overline{S}_D = S_D^* + (1 - f_D^*) / \frac{df_D^*}{dS_D} \qquad (66)$$

So, a straight line represents the relation between R and E_s until water B.T. in the system which increases with decreasing mobility ratio. Also the slope of the straight line, which equals the average saturation, increases with decreasing mobility ratio and approaches unity for the limiting case of zero mobility ratio. As the average saturation increases after B.T., straight line segments with increasing slopes develop as displacement progresses in the succeeding stream tubes.

Fractional Flow Curves
Figure 8 shows the fractional water flow f_w as functions of oil recovery for different values of mobility ratio. Earlier water B.T. occurs at high mobility ratios (unfavorable) and delayed (late) B.T. at low (favourable) mobility ratios. After water B.T. in the first stream tube, f_w exhibits a sudden jump from zero to a value that increases with the increase of the mobility ratio. A slight increase is observed in the value of f_w as displacement proceeds in the successive channels until B.T. occurs in the next tube. At that time a second jump occurs and the behavior is repeated until B.T. occurs in the last stream tube. For unfavorable mobility ratios (M > 1), a sudden decrease is observed as the last cell in each tube is invaded before the jump in f_w occurs at water breakthrough in that tube. This behavior is not observed for favorable mobility ratios (M < 1). It is to be noted that the value of f_w does not reach unity at water B.T. in the last stream tube as would be the case for piston-like displacement.

Injectivity Ratio
As the mobilities of the displaced and displacing fluids are different, the injectivity will vary as more of the displacing fluid enters the reservoir and the injectivity ratio will be different from unity. Since the pressure drop depends on the extent of displacement, injectivity ratio is best correlated with the sweep efficiency. Figure 9 shows the injectivity ratio vs the areal sweep efficiency for different values of mobility ratio. The general behavior observed in Figure 4 occurs for unfavorable mobility ratios (M > 1) where the injectivity ratio increases with time with an abrupt decrease at water B.T. in each stream tube. For favorable mobility ratios (M < 1), the injectivity ratio decreases as displacement progresses and a sharp decrease occurs at time of water B.T. in each of the successive stream tubes. The value of injectivity ratio at the time of water breakthrough in the last stream tube ($E_s = 1$) is not equal to the mobility ratio as supposed to be in piston-like displacement. A modified mobility ratio for the Buckley-Leverett displacement process may be defined as the average between the total mobility at the injection well (λ_w^o) and that at the displacement front λ_t^*.

It was noted that some irregularities occur at the first and last cells in each stream tube. Because Higgins and Leighton divided each tube into 40 equal cells, the first and last cells have the largest geometrical factor (L/A) which mean narrow elongated cells each occupying almost 20% of the total length of the stream tube. A better prediction may be obtained if each of these two cells is divided into a number of smaller cells.

Conclusions

1. A mathematical model is developed to estimate the water flooding performance in 5-spot pattern reservoirs of homogeneous properties. The same procedure can be applied for other flood patterns. The model is based on determining

the location of the displacement front in a stream tubes relative to the front location in another one.

2. Piston-like displacement predicts over optimistic performance as compared to the Buckley-Leverett displacement.

3. The conventional definition of the mobility ratio for piston-like displacement does not apply for B-L displacement.

4. Improved performance prediction may be achieved by increasing the number of stream tubes in the pattern.

Nomenclature

A = area, ft^2 [m^2]
E_s = areal sweep efficiency
f_w = fractional water flow
g = function defined by Eq. (24)
G = geometric factor =L/A
h = formation thickness, ft [m]
I_r = injectivity ratio
K = absolute permeability, md
K_r = relative permeability
L = length, ft [in]
M = mobility ratio
P = pressure, psi [Kpa]
q = flow rate, Bbl/day [m^3/s.]
Q = total flow rate, Bbl/day [m^3/s.]
R = recovery factor, fraction
S = saturation
t = time, s.
V_p = pore volume, ft^3 [m^3]
W = streamtube width, ft [m]
Z = length along stream tube, ft [m]
λ = mobility = k/μ
μ = viscosity, cp [pa.s]
τ = dimensionless time
ϕ = porosity, fraction

Subscripts

D = dimensionless
f = front
i = initial
o = oil
r = releative
t = total
w = water

Superscripts

- = average
* = at breakthrough
o = end point

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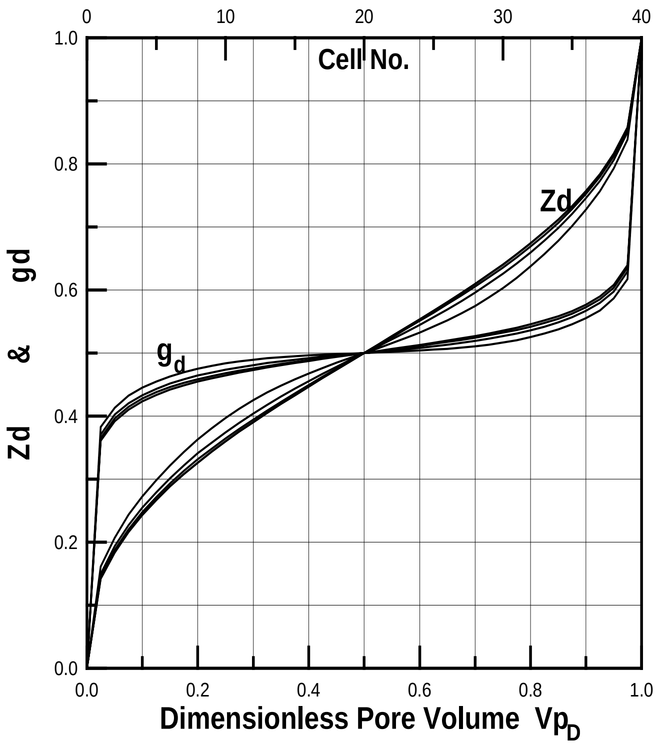

Fig. 1. - Geometrical Characteristics of Stream Tubes

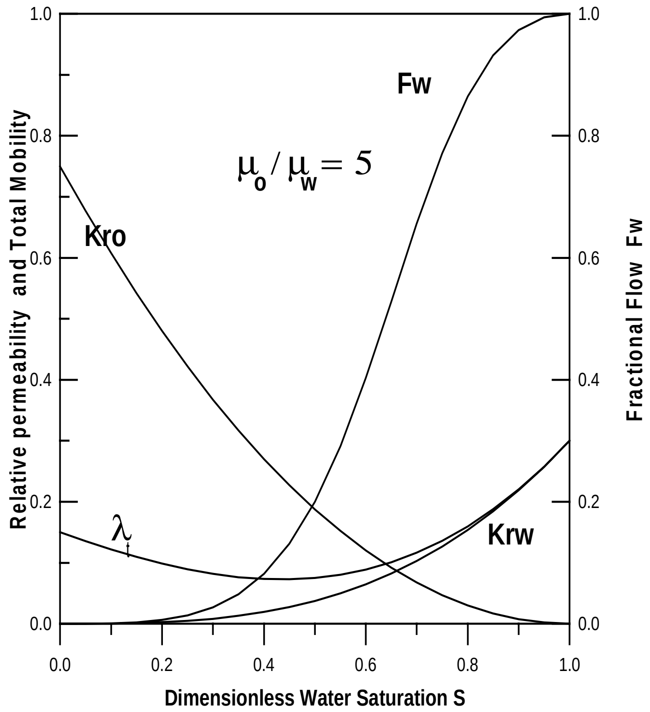

Fig. 2. - Rel. Perm. , Fractional Flow and Total Mobility Curves

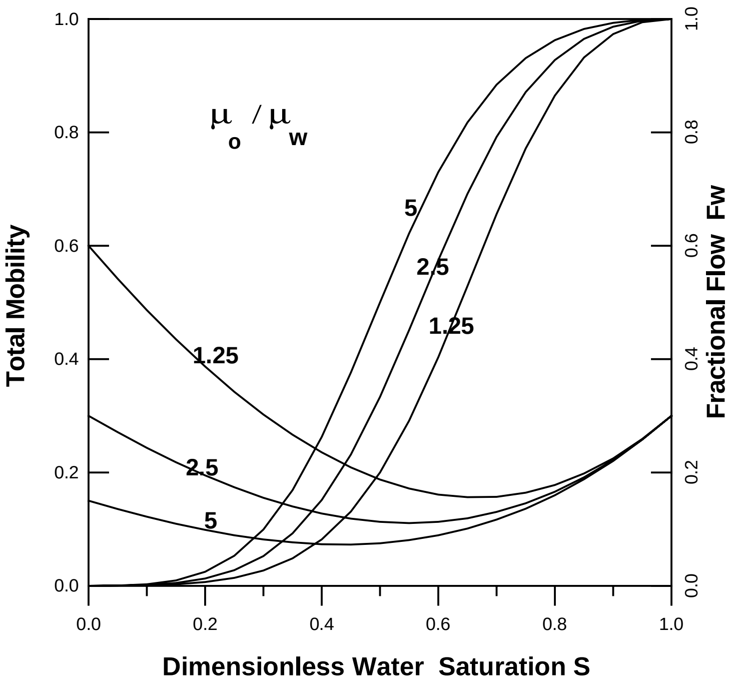

Fig. 3 - Fractional Flow and Total Mobility Curves

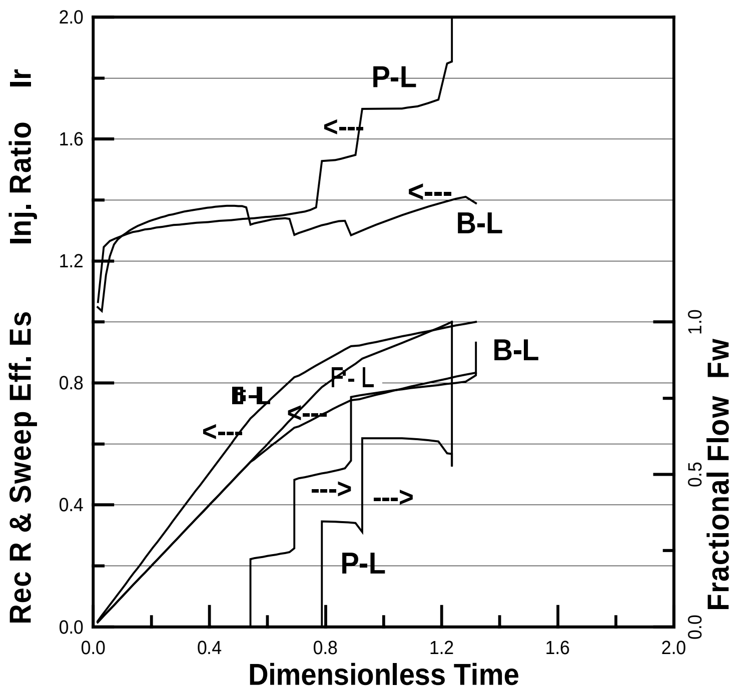

Fig. 4. - Performance Results for Oil/Water Visc. Ratio = 5

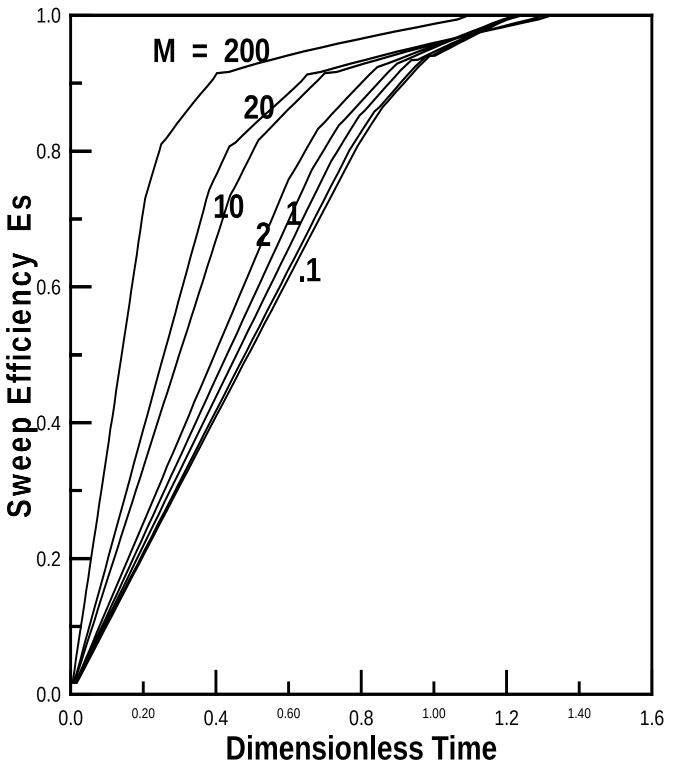

Fig. 5. - Effect of Mobility Ratio on Sweep Efficiency

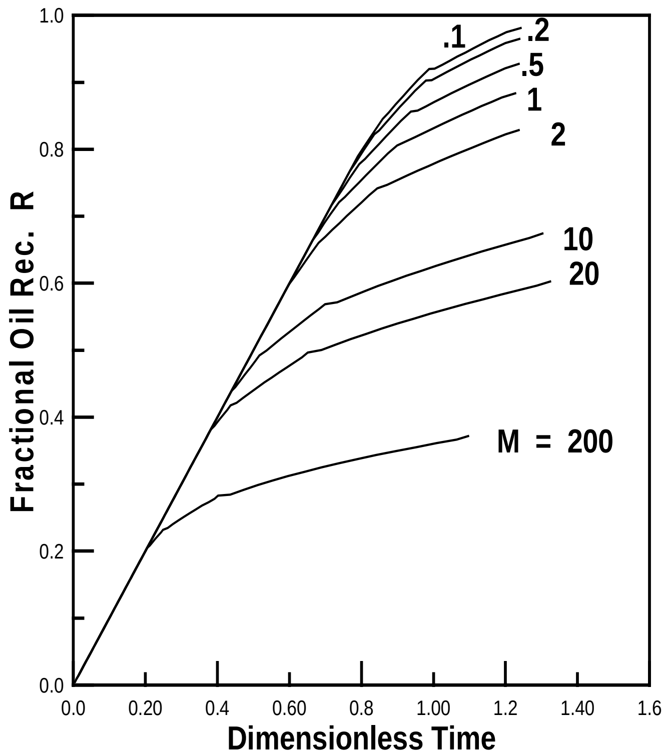

Fig. 6. - Effect of Mobility Ratio on Oil Recovery

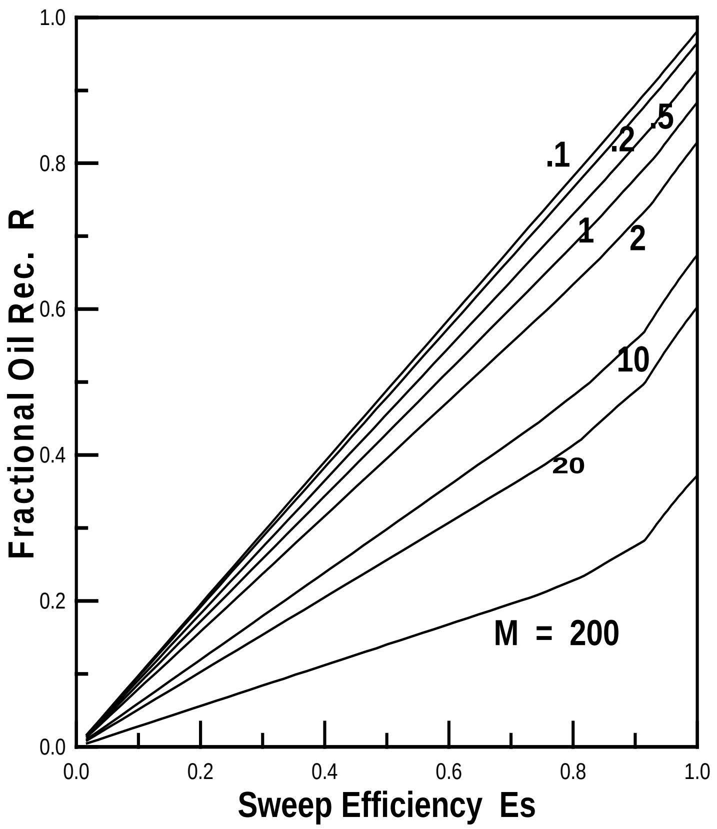

Fig. 7. - Oil Recovery vs. Sweep Efficiency

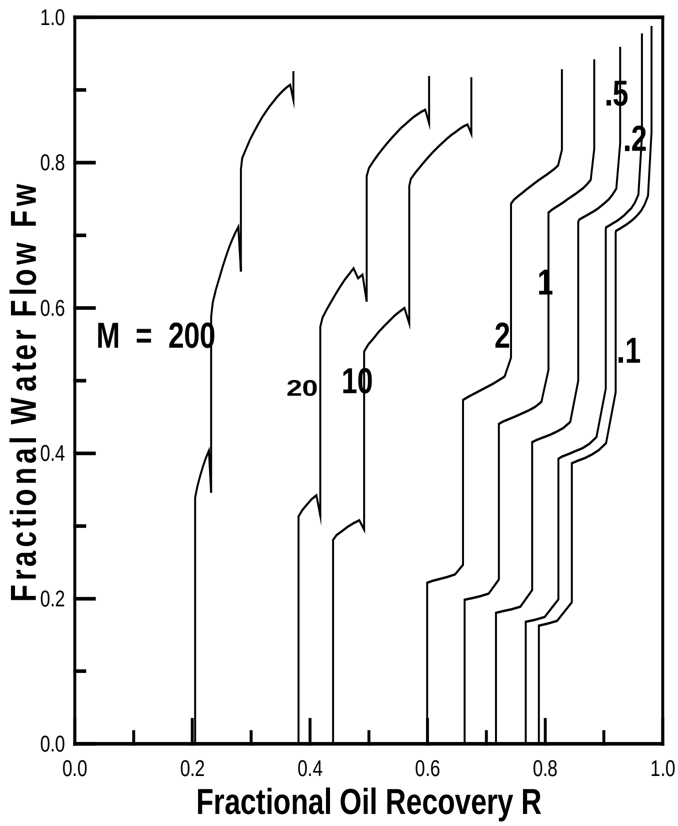

Fig. 8. - Fractional Water Flow vs. Oil Recovery

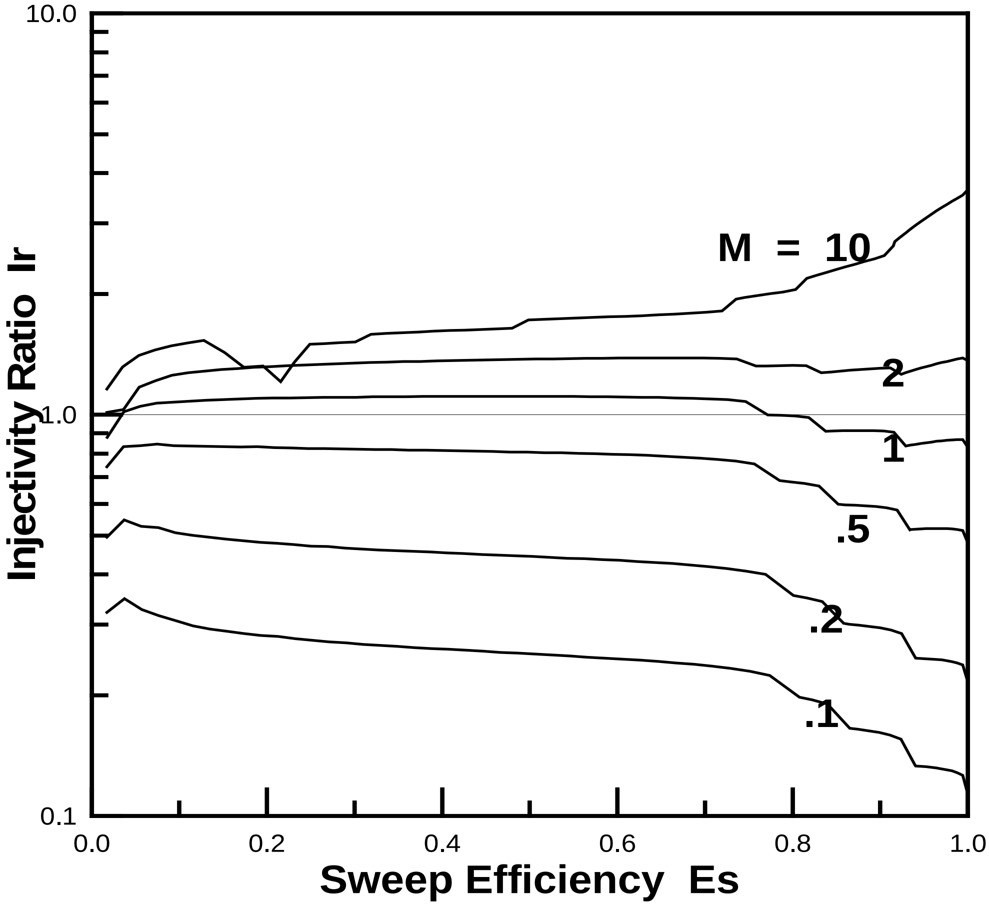

Fig. 9. - Injectivity Ratio vs. Sweep Efficiency