

SPE 29890

Development of a Modified Capillary Pressure J-Function

Noaman El-Khatib*, King Saud University

* SPE Member

Copyright 1995, Society of Petroleum Engineers, Inc.

This paper was prepared for presentation at the SPE Middle East Oil Show held in Bahrain, 11-14 March 1995.

This paper was selected for presentation by an SPE Program Committee following review of information contained in an abstract submitted by the author(s). Contents of the paper, as presented, have not been reviewed by the Society of Petroleum Engineers and are subject to correction by the author(s). The material, as presented, does not necessarily reflect any position of the Society of Petroleum Engineers, its officers, or members. Papers presented at SPE meetings are subject to publication review by Editorial Committees of the Society of Petroleum Engineers. Permission to copy is restricted to an abstract of not more than 300 words. Illustrations may not be copied. The abstract should contain conspicuous acknowledgment of where and by whom the paper is presented. Write Librarian, SPE, P.O. Box 833836, Richardson, TX 75083-3836, U.S.A. Telex 163245 SPEUT.

ABSTRACT

Capillary pressure curves are represented by the equation $P_c = a / (S_w - S_{wi})^b$. It is shown that for the Leverett J-function to produce a single correlation, the tortuosity, the irreducible water saturation and the saturation exponent b must be the same for the different formations. A modified capillary pressure function J^* is introduced that incorporates the tortuosity and irreducible water saturation in its definition. The modified function is correlated with the normalized saturation S_D so that all curves will have their vertical asymptotes at a single point $S_D = 0$. The modified function represents a significant improvement over the original one since it is now sufficient to have the value of the saturation exponent b be the same for two samples to get the same correlation. This was verified by analyzing data from the literature and comparing the two correlations. A modified linear regression by means of weighted least

References and illustration at end of paper

squares method is used to fit the experimental capillary pressure data. Data sets from different sources were analyzed. It was found that a good single correlation can be obtained for samples from the same formation but the correlations were different for different formations.

INTRODUCTION

The capillary phenomena occurs in porous media when two or more immiscible fluids are present in the pore space. Due to the interfacial energy of the interface between the two phases, a difference in the pressure across the interface results and causes a curvature of the interface. The capillary forces causes retention of fluids in the pore space against the gravity forces. Immiscible fluids segregate due to gravity if placed in large containers such as tanks and pipes of large diameters. In porous rocks, however, the dense fluid (water) can be found at higher elevations above the oil-water contact. Capillary pressure is defined as

the difference in pressure between the nonwetting and wetting phases

$$P_c = P_{nw} - P_w \quad (1)$$

Since the gravity forces are balanced by the capillary forces, capillary pressure at a point in the reservoir can be estimated from the height above the oil-water contact and the difference in fluid densities. For an oil-water system

$$P_c = (\rho_w - \rho_o) gh \quad (2)$$

As stated earlier, the difference in pressure causes the interface between the two phases to be curved. The relation between curvature and capillary pressure is given by

$$P_c = \frac{2\sigma \cos\theta}{R} \quad (3)$$

where, R is the average radius of curvature. The curvature of the interface between the two fluids and the shape of the pore space (or of the grains) determine the saturation of the two phases. The relation between the saturation (usually of the wetting phase) and capillary pressure is expressed in terms of a capillary pressure curve or retention curve. If the porous medium is homogenous, i.e. it has the same pore size distribution everywhere, a unique capillary pressure-saturation relation (P_c - S_w curve) exists for the system. Different porous media, however, will have different capillary pressure curves. The shapes of the capillary pressure curves are dependent on the pore space geometry and thus can be used to characterize the porous media.

Capillary pressure curves are usually determined in the laboratory in routine core analysis by the mercury injection method. The determination of capillary pressure using reservoir fluids is usually done by the restored-state method or using a centrifuge. The

mercury injection method is fast and the complete curve for the entire range of saturation is obtained in one run. Other methods are time consuming and a complete run is needed at each saturation. Dynamic measurements of capillary pressures during simultaneous flow of the two immiscible phases were also reported. Comparison between results from the different methods revealed the similarity of the obtained P_c - S_w curves. Data obtained using a certain pair of fluids can be converted for other fluids using the relation

$$\frac{P_{c1}}{P_{c2}} = \frac{(\sigma \cos\theta)_1}{(\sigma \cos\theta)_2} \quad (4)$$

Capillary pressure curves may be determined under drainage or imbibition conditions. Under drainage conditions, a nonwetting phase is displacing a wetting phase, i.e. mercury displacing air or air or gas displacing water or oil. Under imbibition conditions a wetting phase is displacing a nonwetting phase as for the displacement of oil by water in a water wet rock. When repeated cycles of drainage and imbibition tests are performed, hysteresis are observed in the capillary pressure curves. Initially in an oil or gas reservoir underlain by water, equilibrium exists under drainage conditions.

Drainage capillary pressure curves are characterized by an irreducible wetting phase saturation and a threshold or displacement pressure. Threshold pressure, P_d , is the minimum applied pressure needed for the wetting phase to start desaturating. It can be related to the largest pore size according to Eq. (3). The irreducible wetting phase saturation corresponds to the fluid volume retained in very small pores and adsorbed on the surfaces of the solid grains and is not displaced under existing applied pressures. At the irreducible saturation the capillary

pressure curve approaches a vertical asymptote.

Capillary forces as reflected by capillary pressure curves affect the recovery efficiency of oil displaced by water, gas or different chemicals. Capillary pressure functions are thus needed for performing reservoir simulation studies of the different oil recovery processes.

Interpretation of capillary pressure curves can yield useful informations regarding the petrophysical properties of rocks and the fluid rock interaction. Pore size distribution, absolute permeability and relative permeabilities to the wetting and non-wetting phases can be derived from capillary pressure curves if certain simplifying assumptions are made.

Purcell¹ presented relations for estimating absolute and relative permeabilities from capillary pressure curves. Burdine et al² showed how the capillary pressure curves can be used to obtain a pore size distribution function. Holmes and Tippie³ presented a procedure for estimating the contact angle θ at reservoir conditions from laboratory capillary pressure data and saturation vs. depth data from well logs. Thomeer^{4,5} suggested that capillary pressure curves can be represented by a hyperbola on a log-log scale. He correlated the permeability to the geometric parameters of the hyperbola. Swanson⁶ also correlated air and liquid permeabilities with the value of (P_c/S_n) at the point of intersection of a line of 45° with the capillary pressure curve.

Because of the heterogeneity of reservoir rocks, no single capillary pressure curve can be used for the entire reservoir. Attempts were made to correlate capillary pressure curves with the petrophysical properties of the reservoir rocks. Scaling capillary

pressure to the different fluids is achieved by dividing the capillary pressure by the product of the interfacial tension and the cosine of the contact angle ($\sigma \cos \theta$). Leverett⁷ was the first to introduce a dimensionless capillary pressure correlation function. The Leverett J-function is defined as

$$J(S_w) = \frac{P_c}{\sigma \cos \theta} \sqrt{\frac{k}{\phi}} \quad (5)$$

This function which is sometimes termed the universal J-function did not produce a single correlation for the different types of formations. Brown⁸, correlating data for a large number of carbonate cores, concluded that the J-correlation should be used for cores from a single formation and that improvement is obtained if the correlation is restricted to specific lithologic types from the same formation. Rose and Bruce⁹ also presented correlations for 8 different formations showing separate distinguishable curves for the different formations.

Another correlation of capillary pressure data was presented by Guthrie and Greenberger¹⁰. They suggested linear correlation of the water saturation S_w with porosity ϕ and the logarithm of permeability k at a constant value of the capillary pressure.

$$S_w = a_1 \phi + a_2 \log k + c \quad (6)$$

Also quadratic terms of ϕ and $\log k$ were added to Eq. (6) and a better correlation was obtained. Wright and Woody¹¹ applied this correlation to a group of capillary pressure curves of different permeabilities but with constant porosity. Fletcher¹² suggested the use of the Guthrie-Greenberger approach to construct an average capillary pressure curve corresponding to average k and ϕ

from a series of capillary pressure curves of different k and ϕ from the same reservoir.

Alger et al¹³ suggested a linear correlation of the hydrocarbon volume $\phi(1-S_w)$ with the porosity ϕ , permeability k and capillary pressure P_c (or height h)

$$\phi(1 - S_{wi}) = a + b\phi + c \log k + d \log h \quad \dots \quad (7)$$

Unlike the Leverett J-function, the other correlations are merely crossplots for constant values of P_c and do not attempt to normalize the capillary pressure in terms of a dimensionless function. The J-function, however, does not result in a single correlation curve for different formations. Since the J-function scales each capillary pressure curve by multiplying all the points of the curve by the same factor $[(\sqrt{k/\phi}) / (\sigma \cos \theta)]$, individual capillary pressure curves will retain their vertical asymptotes at their respective values of irreducible water saturations. Since the irreducible water saturations is usually not the same for the different formations, it is only logical to expect different J-curves for those formations. Logical reasoning leads us to believe that using a normalized saturation scale may improve the J-correlation. The normalized saturation S_D is defined as

$$S_D = \frac{S_w - S_{wi}}{1 - S_{wi}} \quad (8)$$

Using this definitions, all capillary pressure curves will have the common value of $S_D=0$ at their irreducible water saturation and hence will have the same vertical asymptote.

THEORETICAL CONSIDERATIONS

The flow rate of a fluid in a circular tube of constant radius R is given by the Hagen-Poiseuille formula

$$Q = \frac{\pi R^2 \Delta P}{8 \mu L_e} = \frac{A_f R^2 \Delta P}{8 \mu L_e} \quad (9)$$

where, $A_f = \pi R^2$ is the cross-sectional area of the tube and L_e is the actual length of the tortuous tube.

Fluid flow through porous media is given by the empirical Darcy law.

$$Q = \frac{k A_{tot} \Delta P}{\mu L} \quad (10)$$

where, A_{tot} is the total cross-sectional area (pores + matrix) and L is length of the system.

Equating Q in Eqs. (9) and (10) gives

$$k = \frac{A_f}{A_{tot}} \frac{R^2}{8 \left(\frac{L_e}{L} \right)} \quad (11)$$

since

$$V_p = A_f L_e = A_{tot} L \phi$$

so

$$(A_f / A_{tot}) = \phi / [(L_e / L)] \quad (12)$$

Substituting Eq. (13) into Eq. (11), we get

$$k = \phi \left(\frac{R^2}{8\tau} \right) \quad (13)$$

where, τ is the tortuosity

$$\tau = (L_e / L)^2 \quad (14)$$

Equation (14) is the basis for most of models developed to correlate petrophysical properties of porous media. There are two main approaches of development.

The first approach is that of Kozeny¹⁴ and Carman¹⁵ which uses the concept of hydraulic radius. The hydraulic radius of a conduit is the ratio of the volume to the wetted surface area. For circular tube, the hydraulic radius is half the tube radius. Substituting $R=2R_h$ in Eq. (14)

$$k = \left(\frac{\phi}{2}\right) \left(\frac{R_h^2}{\tau}\right) \quad (15)$$

Kozeny assumed that Eq. (15) would be valid for any porous medium. Since

$$R_h = \phi / S \quad (16)$$

where, S is the specific surface area per unit bulk volume of the rock.

Substituting Eq. (16) into Eq. (15), gives

$$k = \frac{\phi^3}{2\tau S^2} = \frac{\phi^3}{K_z S^2} \quad (17)$$

It has been shown by Dullien¹⁶ that Eq. (15) is not correct for shapes other than circular tubes of constant radius. He showed a deviation of the formula even for circular tubes of unequal diameters connected in parallel or in series. It can be assumed, however, that any deviation can be accounted for by the factor K_z which is called the Kozeny constant. A theoretical value of K_z of 5 was obtained for packings of spheres of constant diameter. However, for actual porous media wide variation of K_z was observed for different formations.

Kozeny equation can also be expressed in terms of surface area per unit grain volume S_g or pore volume S_p . The most widely used form is that in terms of S_g

$$k = \frac{\phi^3}{K_z (1 - \phi)^2 S_g^2} \quad (18)$$

Since S_g depends only on the shape of the grain particles.

The second approach of modeling is that introduced by Purcell¹. The porous medium is assumed to be composed of tubes of different sizes R_i with each size having a fraction of the pore volume ΔS_{wi} . Since the tubes are parallel, the permeability of the system is given by [from Eq. (13)]

$$k = \frac{\phi}{8\tau} \sum_i R_i^2 \Delta S_{wi} \quad (19)$$

Using equation (3) to express the pore radius R_i in terms of capillary pressure P_{ci}

$$k = \frac{\phi}{2\tau} (\sigma \cos \theta)^2 \sum_i R_i^2 \frac{\Delta S_{wi}}{P_{ci}^2} \quad (20)$$

For a continuous system

$$k = \frac{\phi}{2\tau} (\sigma \cos \theta)^2 \int_0^1 R_i^2 \frac{dS_w}{P_c^2} \quad (21)$$

Purcell used a lithology factor λ which is equivalent to $1/\tau$.

Equation (21) represents the theoretical basis for the Leverett J -correlation. It can be written in the following form

$$\int_0^1 \frac{dS_w}{\left[\frac{P_c}{\sigma \cos \theta} \sqrt{\frac{k}{\phi}} \right]^2} = \int_0^1 \frac{dS_w}{J^2(S_w)} = 2\tau \quad \dots \quad (22)$$

Equation (22) reveals that the J-function as proposed by Leverett is not unique for all porous media. The integral of the reciprocal of the square of the function over the saturation range is dependent on the tortuosity which will be different for the different formations. This equation gives a clue to the modification of the J-function. Inclusion of the tortuosity in the definition of the function lets the integral has a value of 2 for all formations.

Equation (22) can also be used to evaluate the tortuosity τ

$$\tau = \frac{1}{2} \int_0^1 \frac{dS_w}{J^2(S_w)} \quad (23)$$

DEVELOPMENT OF THE MODIFIED J-FUNCTION

Since capillary pressure curves have a vertical asymptote at the irreducible water saturation S_{wi} , the shape of the curve suggests the following empirical relation

$$P_c = \begin{cases} \frac{a}{(S_w - S_{wi})^b} & S_w > S_{wi} \\ \infty & S_w < S_{wi} \end{cases} \quad (24)$$

Substituting Eq. (24) for P_c in Eq. (21) to evaluate the absolute permeability,

$$k = \frac{\phi}{2\tau} \frac{(\sigma \cos \theta)^2}{a^2} \int_{S_{wi}}^1 (S_w - S_{wi})^{2b} dS_w \quad \dots \quad (25)$$

Since $1/P_c^2 = 0$ for $S_w < S_{wi}$, the integration is carried out between S_{wi} and 1. Evaluating the integral in Eq. (25) and rearranging we get

$$\frac{1}{\sigma \cos \theta} \sqrt{\frac{k}{\phi}} = \frac{1}{\sqrt{2\tau} a} \frac{(1 - S_{wi})^{(2b+1)/2}}{\sqrt{2b+1}} \quad \dots \quad (26)$$

Dividing Eq. (24) by Eq. (26), we get

$$\frac{P_c}{\sigma \cos \theta} \sqrt{\frac{k}{\phi}} = \frac{1}{\sqrt{2\tau}} \frac{1}{\sqrt{2b+1}} \frac{(1 - S_{wi})^{b+0.5}}{(S_w - S_{wi})^b} \quad \dots \quad (27)$$

The left hand side of Eq. (27) is the Leverett J-function. So we have

$$J = \frac{1}{\sqrt{2\tau}} \cdot \frac{1}{\sqrt{2b+1}} \frac{\sqrt{(1 - S_{wi})}}{\left(\frac{S_w - S_{wi}}{1 - S_{wi}} \right)} \quad \dots \quad (28)$$

Equation (28) shows that the J-function is not a unique function of the saturation S_w . It depends on the two parameters S_{wi} and τ . If the normalized saturation defined by Eq. (8), S_D , is used, Eq. (28) can be rearranged in the following form

$$J = \sqrt{\frac{\tau}{1 - S_{wi}}} = \frac{\left(\frac{1}{\sqrt{4b+2}} \right)}{S_D^b} \quad (29)$$

If the left hand side of Eq. (29) is considered as a modified capillary pressure correlation J^* we can write

$$J^*(S_D) = B / S_D^b \quad (30)$$

where,

$$J^* = \frac{P_c}{\sigma \cos \theta} \sqrt{\frac{k\tau}{\phi(1-S_{wi})}} \quad (31)$$

and

$$B = (1 / \sqrt{4b + 2}) \quad (32)$$

A necessary and sufficient condition for the $J^*(S_D)$ to be unique is that the saturation exponent b to be the same for different formations. If b is constant, then the coefficient B will also be constant according to Eq. (32). It can be seen that the modified correlation J^* represents a considerable improvement over the function J which requires S_{wi} and τ in addition to the saturation exponent b to be the same for all formations.

CONSTRUCTIONS OF THE $J^*(S_D)$ CORRELATION

To convert a capillary pressure curve (P_c vs. S_w) to the $J^*(S_D)$ one needs the values of k , ϕ , S_{wi} , τ and ($\sigma \cos \theta$) for the curve under consideration. Equation (23) can be used to evaluate τ and the irreducible water saturation may be obvious from the capillary pressure curve. In most capillary pressure curves, values of P_c at saturations S_w close to unity (including the threshold pressure) are difficult to determine. Investigation of Eq. (23) shows the sensitivity of the result (value of τ) to data in that region. Actually most of the contribution to the permeability k from the integral in Eq. (21) is due to the small values of P_c (large pores) where $(1/P_c)$ is very large. In addition to this, values of the irreducible water saturation S_{wi} may not be reached during the capillary pressure experiments. For these reasons, capillary pressure data points are used to fit Eq. (24) to obtain values of a , b and S_{wi} . A weighted linear least squares method is used to obtain the best fit of the data. This procedure is outlined in Appendix (A).

The values of a , b and S_{wi} obtained from the curve fitting are used with the values of k , ϕ and $\sigma \cos \theta$ in Eq. (26) to estimate the tortuosity τ

$$\tau = \frac{(\sigma \cos \theta)^2}{a^2} \left(\frac{\phi}{k} \right) \frac{(1 - S_{wi})^{2b+1}}{(4b + 2)} \quad \dots \quad (33)$$

From Eq. (27) it can be seen that

$$\begin{aligned} J^* &= \frac{P_c}{\sigma \cos \theta} \sqrt{\frac{k\tau}{\phi(1-S_{wi})}} \\ &= C P_c \end{aligned} \quad (34)$$

where,

$$C = \frac{(1 - S_{wi})^b}{a\sqrt{4b + 2}} \quad (35)$$

It is seen that values of a , b and S_{wi} obtained from fitting P_c-S_w data can be used directly to estimate $J^*(S_D)$ without evaluating the tortuosity τ and even without the knowledge of the porosity and permeability of the rock sample or the properties of the fluids used. This is apparently due to the fact that all these informations are embedded in the capillary pressure curve. This is also clear from Eq. (26) or (33) where the value of C given by

$$C = \frac{1}{\sigma \cos \theta} \sqrt{\frac{k\tau}{\phi(1-S_{wi})}} \quad (36)$$

is determined completely from the parameters of the capillary pressure curve, i.e. a , b and S_{wi} .

RESULTS AND DISCUSSION

More than 50 capillary pressure curves were analyzed. For each curve the data were fitted according to Eq.

(24) and the modified as well Leverett capillary pressure functions were calculated. To compute the Leverett function, the porosity, ϕ , permeability, k , and interfacial tension and contact angle ($\sigma \cos \theta$) are needed. To compute the modified function J^* , the values of these parameters are not needed. These values are needed, however, if the tortuosity τ is to be calculated. Table (1) shows the results for one of the capillary pressure curves.

On a log-log graph, the modified function J^* plots a straight line with intercept B (at $S_D=1$) and slope $-b$. When the data for all the capillary pressure curves from different formations are plotted without sorting on a single plot, the results are shown in Figure 1. On the same plot, lines for ideal cases of different values of b are also shown. The wide scatter of the data points indicates that a single (universal) correlation is not possible and that data should be sorted for the different formations. The average value of b for the all data is 1.077.

In the next step the data given by Purcell were considered. Figure 2 shows the seven capillary pressure curves from four different formations. A summary of the characteristics of the formations and results obtained from the analysis are shown in Table (2). Plots of the Leverett capillary pressure J -function and the modified J^* function are shown in Figures 3 and 4 respectively. Comparison of the two figures shows wide scattering of the J -function as compared to the J^* correlation. Since the correlation represents four different formations, it could be improved if each formation is taken separately. Such separation, however, was not tried because enough samples were not available for each formation (3, 2, 1 and 1). Nevertheless, Figure 4 shows a good correlation for the combined data of the four formations.

Another set of capillary pressure curves correlated are those of Schnaupp gas field in West Germany¹⁹. A set of capillary pressure curves for this carbonate reservoir were given for different porosities. Since the permeabilities were not given, it was not possible to calculate the Leverett function J . The modified function J^* was computed by the procedure developed here and the results of five different curves with porosities ranging between 6% and 22% are shown in Figure 5. The figure shows the excellent correlation of the data.

Correlation for eight capillary pressure curves measured at King Saud University (KSU) on synthetic sandstone cores²⁰ are shown in Figure 6. Although the cores have different percentages of quartz indicating different pore size distributions, the correlation is seen to be good. Figure 7 shows the correlation of the Leverett function for the same data. Comparison of the two figures indicates the superiority of the modified function.

Values of the capillary pressure exponent b for the curves considered in this study ranged between 0.5 and 5 with an average value of 1.077. Some investigators²¹ suggested the equivalence of b with the saturation exponent n relating the resistivity index I to the formation water saturation S_w according to Archie equation

$$I = \frac{R_t}{R_o} = S_w^{-n} \quad (37)$$

This is a consequence of the observation of constant resistivity gradient dR_t/dh in the transition zone above the O/W contact for clean formation. A value of $n=2$ is widely accepted in log interpretation practices. Eleven curves for which both capillary pressure and resistivity measurements are availa-

ble^{22,23} gave good agreement on the average between b and n with an average value of about 2.0. Figure 8 shows the correlation of b and n for the data. These results, however, cannot be generalized. Wyllie and Rose²⁴ suggested an average value of 0.5 for b . They also suggested that the value of n in the Archie equation is not constant for the entire saturation range. This suggestion was recently verified by the experimental work of Soendenaa et al²². The value of the capillary pressure exponent b may also be saturation dependent. This behavior may be seen from the log-log plots of Figure 1 for the entire data set and Figure 9 for KSU data where the data points exhibit a curvature increasing with the increase of the water saturation.

The two additional variables, τ and S_{wi} in the definition of the modified capillary pressure function $J^*(S_D)$ are not directly measurable from well logs. It is also to be noted that the value of S_{wi} obtained from fitting the P_c-S_w data to Eq. (24) is not the same as that read from the capillary pressure curves or determined in the lab. The maximum pressure applied in most tests does not reach that needed to establish the irreducible water saturation. For the practical application of the developed function J^* , correlations must be found for τ and S_{wi} in terms of the other petrophysical properties, i.e. porosity ϕ and permeability k . It was suggested that the tortuosity τ is related to porosity by the relation

$$\tau = F \phi \quad (38)$$

where F is the resistivity formation factor which in turn is a function of porosity. The irreducible water saturation S_{wi} is a function of the specific surface area S which according to Kozeny equation relates the porosity, permeability and tortuosity. This

suggests that S_{wi} can also be related to k and ϕ . Such relations were suggested by different investigators^{24,25} but no general relation is applicable for all formations. If valid relations can be developed for a particular formation, these relations can be used to estimate permeability from log derived values of porosity ϕ and water saturation S_w at a given location in the well. The capillary pressure at the well is estimated according to Eq. (2). The tortuosity τ is estimated from the ϕ - τ relation and a trial and error procedure (iteration) can be used to estimate S_{wi} and k from the J^* function and the S_{wi} relation with both ϕ and k .

As it was noted before, if two capillary pressure curves have the same saturation exponent b , they will have the same function J^* . This suggests the possibility of correlating different zones in a formation in the different wells according to the value of the saturation exponent b as determined from capillary pressure curves measured on cores taken from the different wells.

CONCLUSIONS

1- The Leverett capillary pressure function $J(S_w)$ requires for a unique correlation that the tortuosity τ , the irreducible water saturation S_{wi} and the saturation exponent b be the same for the different formations.

2- A modified capillary pressure function $J^*(S_D)$ is developed that incorporates the tortuosity and irreducible water saturation in its definition. This function requires only the equality of the saturation exponent b in the different formations to obtain a unique correlation. Results showed the superiority of the J^* correlation to the J correlation.

3- To convert capillary pressure data (P_c vs. S_w) to the $J^*(S_D)$ function is independent of the values of porosity, permeability and interfacial tension. The combination of these parameters needed to estimate J^* are derived from the capillary pressure curves.

4- If relations can be obtained for irreducible water saturation S_{wi} and tortuosity τ in terms of porosity and permeability for a given formation, the modified function correlation J^* can be used to estimate permeability from log derived values of ϕ and S_w .

5- It may be possible to use the capillary pressure exponent b in correlating zones in different wells.

NOMENCLATURE

a	= constant, Eq (24), Pa
A	= cross-sectional area, m^2
b	= saturation exponent, Eq. (24)
B	= constant, Eq. (32)
C	= constant, Eq. (35)
h	= height above O/W contact, m
$J(S_w)$	= Leverett - capillary pressure function, Eq. (5)
J^*	= modified capillary pressure function, Eq. (31)
k	= permeability, m^2
K_z	= Kozeny constant
L	= length, m
L_e	= tortuous length, m
P	= pressure, Pa
P_c	= capillary pressure, Pa
ΔP	= pressure drop, Pa
Q	= flow rate, m^3/sec
R	= pore entry or tube radius
S	= specific surface area, m^{-1}
S_D	= normalized saturation, Eq. (8)
S_w	= water or wetting phase saturation, fraction
S_{wi}	= irreducible water saturation
θ	= contact angle
ρ	= density, Kg/m^3
σ	= interfacial tension, Nm

τ = tortuosity, Eq. (14)

ϕ = porosity, fraction

SUBSCRIPTS

f = flow

g = grain

nw = nonwetting

o = oil

t = total

w = water or wetting

REFERENCES

1. Purcell, W.R., "Capillary Pressures - Their Measurements Using Mercury and the Calculation of Permeability Therefrom," *Trans. AIME* (1949) **186**, 39-48.
2. Burdine, N.T., Gournay, L.S., and Reichertz, P.P., "Pore Size Distribution of Petroleum Reservoir Rocks," *Trans. AIME* (1950) **189**, 195-204.
3. Holmes, Michael and Tippie, Douglas, B., "Comparison Between Log and Capillary Pressure Data to Estimate Reservoir Wetting," paper SPE 6856, presented at the 52nd Annual Fall Tech. Conf. and Exhib. of SPE, Denver, Colorado, Oct. 9-12, 1977.
4. Thomeer, J.H.M., "Introduction of a Pore Geometrical Factor Defined by the Capillary Pressure Curve," *JPT*, (March 1960) 73-77.
5. Thomeer, J.H.M., "Air Permeability as a Function of Three Pore-Network Parameters," *JPT* (April 1983) 809-14.
6. Swanson, B.F., "A Simple Correlation Between Permeabilities and Mercury Capillary Pressures," *JPT* (Dec. 1981) 2498-504.

7. Leverett, M.C., "Capillary Behavior in Porous Solids," *Trans. AIME* (1941) **142**, 152-69.
8. Brown, Henry W., "Capillary Pressure Investigations," *Trans. AIME* (1951) **192**, 67-74.
9. Rose, Walter and Bruce, W.A., "Evaluation of Capillary Characters in Petroleum Reservoir Rocks," *Trans. AIME* (194) **186**, 127-142.
10. Amyx, J.W., Bass, D.M. and Whiting, R.L., "*Petroleum Reservoir Engineering*, McGraw-Hill, Book Co., Inc., New York City (1960) 158-61.
11. Wright, H.T., Jr., and Woody, L.D., Jr., "Formation Evaluation of the Borregas and Seeligson Field, Brooks and Jim Wells Country, Texas," Symposium on Formation Evaluation, AIME, October, 1955.
12. Pletcher, Jeffry L., "A Practical Capillary Pressure Correlation Technique," *JPT* (July 1994) 556.
13. Alger, R.P., Luffel, D.L., and Truman, R.B., "New Unified Method of Integrating Core Capillary Pressure Data," paper SPE 16793 presented at the 62nd Annual Tech. Conf. and Exhib. of SPE, Dallas, TX, Sept. 27-30, 1987.
14. Kozeny, J., "Über Kapillare Leitung des Wassers im Boden," *Sitzungsber. Akad. Wiss. Wien* (1927) **136**, 271-306.
15. Carman, P.C., "Capillary Rise and Capillary Movement of Moisture in Fine Sands," *Soil Sci.*, (1941) **52**, 1-14.
16. Dullien, Francis, A.L., "New Network Permeability Model of Porous Media," *AICHE J.* (March 1975) **21**, 2, 299-307.
17. Maute, R.E., Lyle, W.D. and Sprunt, Eve S., "Improved Method Determines Archie Parameters From Core Data," *JPT* (Jan. 1992) 103-107.
18. Day, Peter I., "Discussion of Improved Data-Analysis Determines Archie parameters From Core Data," *JPT* (July 1992) 846.
19. Rieckmann, Martin, "Evaluation of Reservoir Rocks of Low Permeability," (in German), presented at the 6th World Petroleum Congress, Frankfurt/Main, West Germany (1963), published in *Erdöl und Kohle, Erdgas, Petrochemie*, special issue, (June 1963) - Section II, 488-496.
20. Al-Balawi, Yousif, S., "Petrophysical Measurements on Consolidated Sandstone Cores," Unpublished B.Sc. Project, King Saud University, Riyadh (1994).
21. Tixier, M.P., "Evaluation of Permeability from Electric-log Resistivity Gradient," *Oil and Gas J.* (June 16, 1949) **48**, NO. 6, 113-122.
22. Soendenna, E., Bratteli, F., Kolltveit, K. and Normann, H.P., "A Comparison Between Capillary Pressure Data and Saturation Exponent Obtained at Ambient Conditions and at Reservoir Conditions," paper SPE 19592 presented at the 64th Annual Tech. Conf. and Exhib. of SPE, San Antonio, TX., Oct. 8-11, 1989.
23. Longeron, D.G., Argaud, M.J., and Bouvier, L., "Resistivity Index and Capillary Pressure Measurements Under Reservoir Conditions Using Crude Oil," paper SPE 19589 presented at the

64th Annual Tech. Conf. and Exhib. of SPE, San Antonio, TX., Oct. 8-11, 1989.

- 24. Wyllie, M.R.J. and Rose, W.D., "Some Theoretical Considerations Related to the Quantitative Evaluation of the Physical Characteristics of Reservoir Rock from Electrical Log Data," Trans. AIME (1950) **189**, 105-118.
- 25. Timur, A., "An Investigation of Permeability, Porosity, and Residual Water Saturation Relationships for Sandstone Reservoirs," *The Log Analyst* (July-Aug. 1968) **9**, No. 4

APPENDIX — Regression Analysis of Capillary Pressure Data by Weighted Linear Least Squares Method

It is required to fit the capillary pressure data using the equation

$$P_c = \frac{a}{(S_w - S_{wi})^b} \tag{A-1}$$

The method of least squares is based on finding values of the parameters a, b and S_{wi} that minimizes the sum of squares of the difference between observed and calculated capillary pressures. The objective function F to be minimized is given by

$$F = \sum_j [P_{c\text{ ob}} - P_{c\text{ cal}}]^2$$

$$= \sum_j \left[P_{c j} - \frac{a}{(S_w - S_{wi})_j^b} \right]^2 \tag{A-2}$$

Since the terms of Eq. (A-2) are nonlinear, nonlinear regression analysis is needed to find a, b and S_{wi} . This requires iteration and convergence is not guaranteed unless the initial guess of

the parameters is very close to the correct solution.

The nonlinear optimization problem can be linearized by writing Eq. (A-1) in a logarithmic form

$$\log P_c = a_1 + b_1 \log(S_w - S_{wi}) \tag{A-3}$$

where,

$$a_1 = \log a \text{ and } b_1 = -b$$

To find a_1 , b_1 and S_{wi} we need to minimize the function

$$F_1 = \sum [\log P_c - (a_1 + b_1 \log(S_w - S_{wi}))]^2 \tag{A-4}$$

The parameter S_{wi} is still causing nonlinearity. This can be handled by assuming successive values of S_{wi} and determining the values of a_1 and b_1 that minimize F_1 . The value of S_{wi} that gives the minimum value of F_1 is chosen.

Although the problem can be linearized by transforming it to the logarithmic space, many authors^{17,18} stress the need to optimize in the original space. It would be advantageous if the differences in Eq. (A-2) can be expressed in terms of those in Eq. (A-4). This would combine the accuracy of the nonlinear regression with the simplicity of the linear regression. This is achieved as follows:

$$\Delta \log P_c = \log P_{c\text{ cal}} - \log P_{c\text{ ob}}$$

$$= \log \left(\frac{P_{c\text{ cal}}}{P_{c\text{ ob}}} \right) = \log \left(\frac{P_{c\text{ ob}} + \Delta P_c}{P_{c\text{ ob}}} \right)$$

$$= \log \left(1 + \frac{\Delta P_c}{P_{c\text{ ob}}} \right) \tag{A-5}$$

where,

$$\Delta P_c = P_{c\text{ cal}} - P_{c\text{ obs}}. \tag{A-6}$$

Expanding Eq. (A-5) by Taylor's series

$$\Delta \log P_c = \log(1) + \frac{\Delta P_c}{P_{c\text{ob}}} + \frac{1}{2} \left(\frac{\Delta P_c}{P_{c\text{ob}}} \right)^2 + \dots \quad (\text{A-7})$$

Neglecting second order terms of $(\Delta P_c/P_{c\text{obs}})$ and higher and since $\log(1)=0$, we get

$$\Delta \log P_c = \Delta P_c / P_{c\text{ob}}$$

or

$$\Delta P_c = P_{c\text{obs}} \Delta \log P_c \quad (\text{A-8})$$

Substituting Eq. (A-8) into (A-2)

$$F = \sum P_{cj}^2 [\log P_{cj} - a_1 - b_1 \log(S_w - S_{wi})]^2 \quad (\text{A-9})$$

Eq. A-9) may be looked at as a weighted linear least squares method where a weighting facator P_{cj}^2 is allocated for the j th observation.

The foregoing derivation can be simply deduced from the fact that

$$d(\ln y) = dy / y \quad (\text{A-10})$$

and so

$$dy = y d(\ln y) \quad (\text{A-11})$$

The values of a_1 and b_1 that minimize F in Eq. (A-9) for a given value of S_{wi} are given by

$$\begin{vmatrix} \sum w_j & \sum w_j x_j \\ \sum w_j x_j & \sum w_j x_j^2 \end{vmatrix} \begin{vmatrix} a_1 \\ b_1 \end{vmatrix} = \begin{vmatrix} \sum w_j y_j \\ \sum w_j x_j y_j \end{vmatrix} \quad \dots \quad (\text{A-12})$$

where, $w = P_c^2$, $y = \log P_c$
and $x = \log(S_w - S_{wi})$

Table 1: Curve Fitting and Calculation of Capillary Pressure Functions*

S_w	P_c	$P_{c\text{fit}}$	J_1	S_D	J^*
0.37	8.00	8.000	2.505	.0597	4.249
0.41	4.56	4.573	1.428	.1194	2.422
0.48	2.78	2.754	0.870	.2239	1.477
0.54	2.15	2.099	0.676	.3134	1.147
0.61	1.64	1.665	0.513	.4179	0.871
0.65	1.40	1.495	0.438	.4776	0.744
0.70	1.30	1.329	0.407	.5522	0.691
0.80	1.15	1.096	0.360	.7015	0.611

Parameters From Curve Fitting

$$\begin{aligned} a &= 0.5961 & \tau &= 3.856 \\ b &= 0.8067 & B &= 0.4374 \\ S_{wi} &= 0.33 & C &= 0.5312 \end{aligned}$$

* Data for a Frio Sandstone Formation, Purcell curve no. 3

Table 2: Summary of Results for Purcell's Capillary Pressure Data

No.	Formation characteristics					Results from curve fitting			
	Formation	Fig.*	K(md)	ϕ	τ_1^{**}	a	b	S_{wi}	τ
1	Frio S.S	2	23	0.23	13.7	0.427	1.188	0.505	10.11
2	Frio S.S	3	170	0.26	3.69	0.596	0.807	0.33	3.86
3	Frio S.S	4	950	0.25	3.62	0.153	1.668	0.21	6.24
4	Paluxy S.S	5	1150	0.20	2.95	0.142	1.462	0.01	14.02
5	Wilcox S.S	6	430	0.22	4.63	0.287	1.410	0.04	9.30
6	San Andres LS	7	35	0.19	7.52	1.409	0.768	0.00	7.19
7	San Andres LS	8	43	0.16	6.67	1.060	0.780	0.075	7.06

* Fig no. in Purcell's paper
 ** From Purcell's paper = 1/F

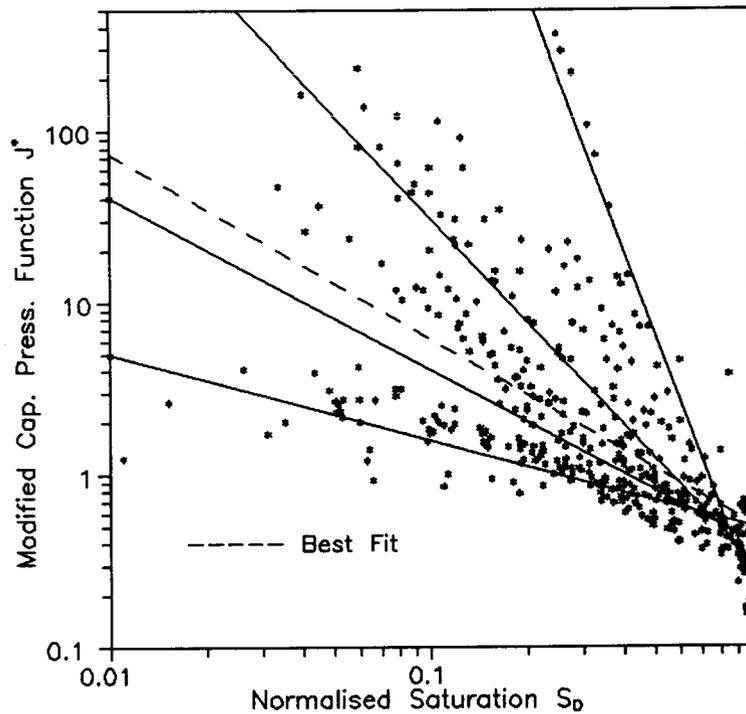


Fig. 1 - Log - Log Plot for All Data

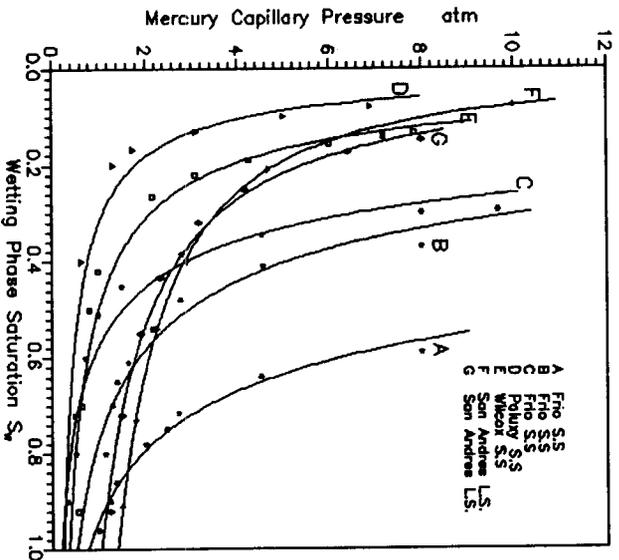


Fig. 2 — Capillary Pressure Curves — Purcell's Data

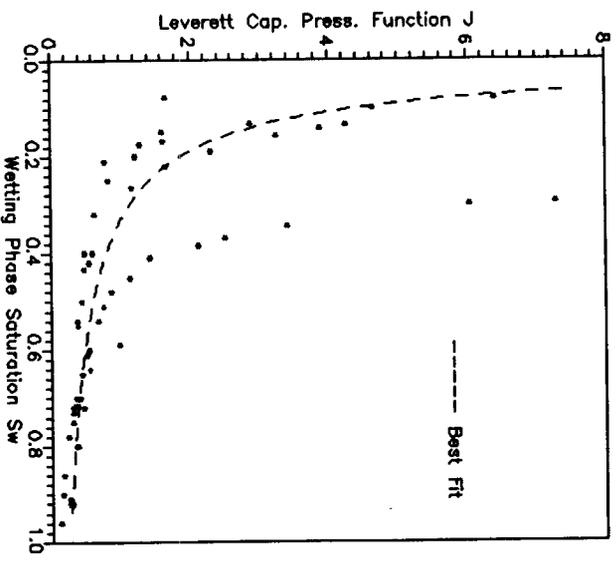


Fig. 3 — Leverett Function for Purcell's Data

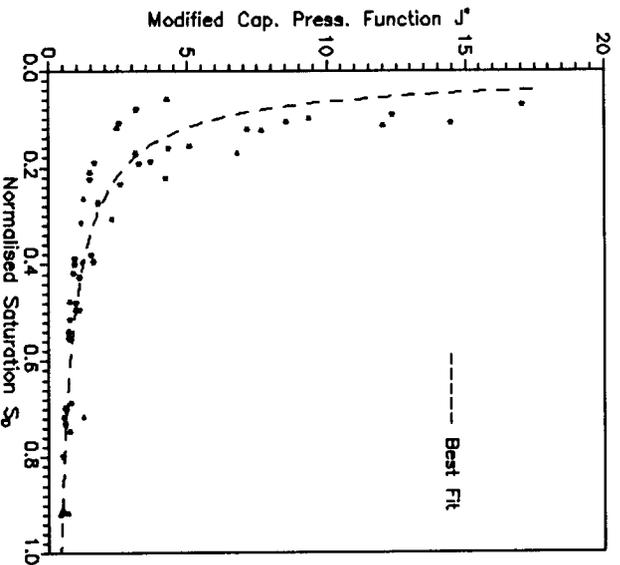


Fig. 4 — Modified Function for Purcell's Data

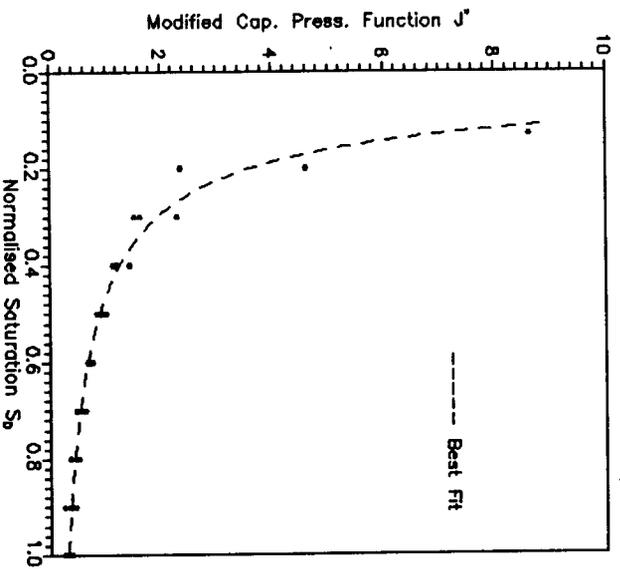


Fig. 5 — Modified Function for Schnauppig Field

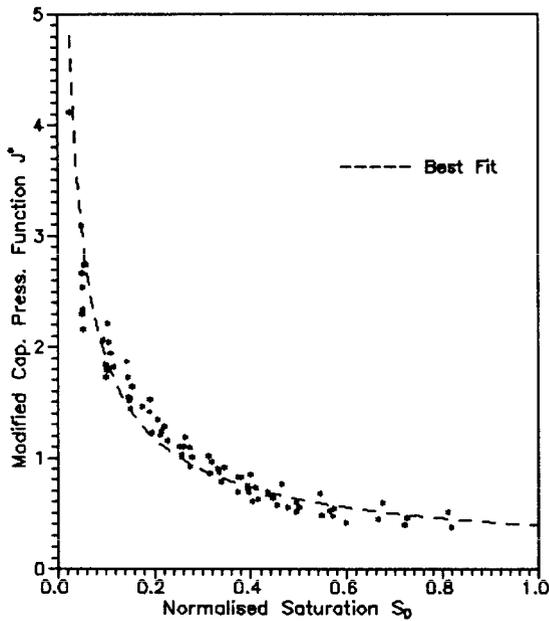


Fig. 6 - Modified Function for KSU Data

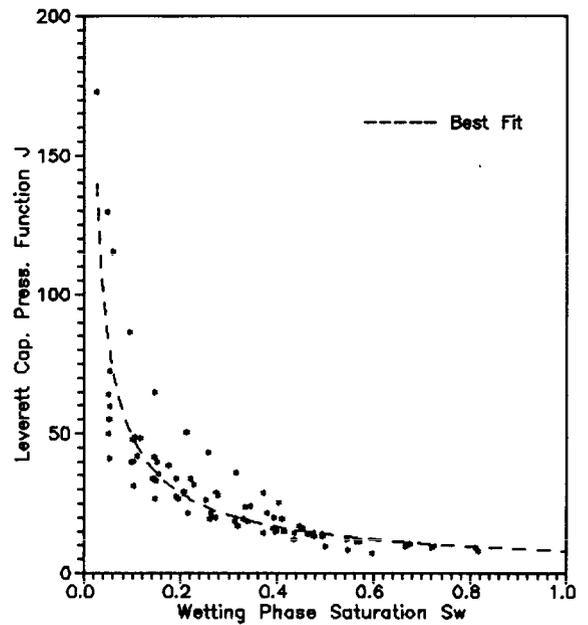


Fig. 7 - Leverett Function for KSU Data

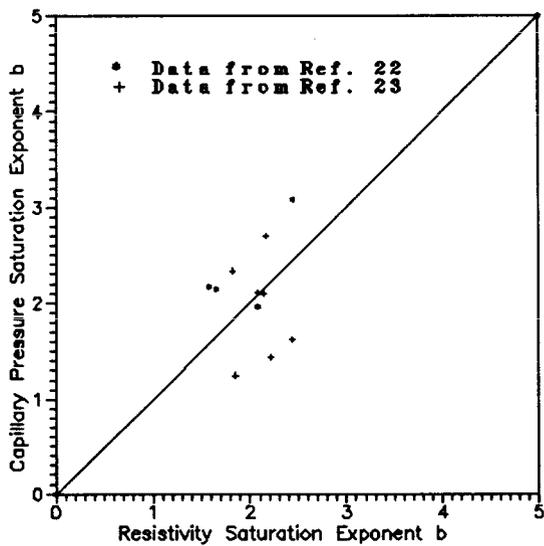


Fig. 8 - Comparison between b and n Exponents

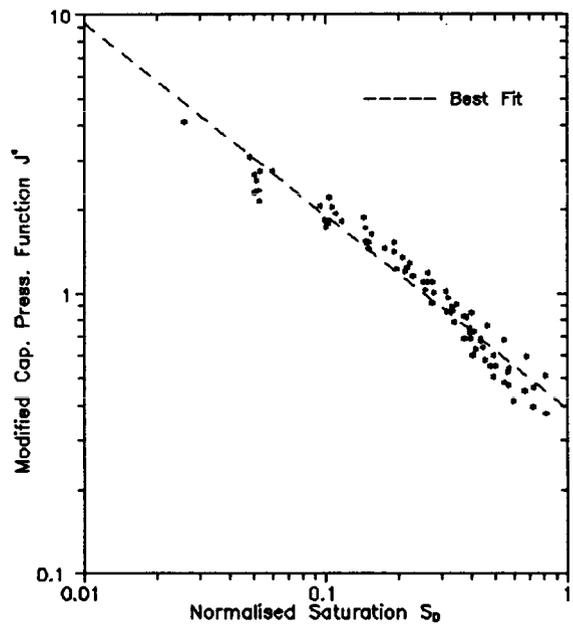


Fig. 9 - Log - Log Plot for KSU Data