The Effect of Crossflow on Waterflooding of Stratified Reservoirs

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Abstract

A mathematical model is developed for waterflooding performance in linear stratified systems for both cases of noncommunicating layers with no crossflow and communicating layers with complete crossflow. The model accounts for variation of porosity and saturation in addition to permeability of the different layers. The model predicts the fractional oil recovery, the water cut, the total volume injected, and the change in the total pressure drop, or the change in injection rate at the water breakthrough in the successive layers. A systematic procedure for ordering of layers and performing calculations is outlined. A procedure for combining layers to avoid instability in the case of low mobility ratio is introduced.

The developed model is applied to different examples of stratified reservoirs. The effects of mobility ratio and crossflow between layers are discussed. The effects of variable porosity and fluid saturation are discussed also.

It was found that crossflow between layers enhances the oil recovery for systems with favorable mobility ratios ($\lambda_w/\lambda_o < 1$) and retards oil recovery for systems with unfavorable mobility ratios. It was found also that crossflow causes the effect of the mobility ratio on oil recovery to become more pronounced. The variation of porosity and fluid saturation with permeability is found to increase oil recovery over that for the case of uniform porosity and saturation for both favorable and unfavorable mobility ratios.

Introduction

Because of the variation in the depositional environments, oil-bearing formations usually exhibit random variations in their petrophysical properties in both horizontal and vertical directions. Statistical as well as geological criteria[1,2] usually are used to divide the pay zone between adjacent wells into a number of horizontal layers each with its own properties (k, ϕ, h, S_{wi}, and S_{or}). Such reservoirs usually are called "stratified," "layered," or "heterogeneous" reservoirs.

This variation in properties affects the performance of oil reservoirs during primary and secondary recovery processes. One of the significant factors influencing recovery performance during waterflooding is the variation of permeability in the vertical direction. In this case, the displacing fluid (water) tends to move faster in zones with higher permeabilities, causing earlier breakthrough of water into the producing wells and eventual bypassing of some of the displaced fluid (oil).

The various methods[3–8] used for the prediction of waterflooding performance of stratified reservoirs differ in the way the communication between the different layers is treated. Two ideal cases usually are used: (1) completely noncommunicating layers and (2) communicating layers with complete crossflow. For actual stratified systems, however, the layers are partially connected in the vertical direction, and the performance of the system lies between those of the two ideal cases.

For the case of noncommunicating stratified layers, the methods of Stiles[3] and Dykstra-Parsons[4] usually are used. Stiles' method assumes unit mobility ratio for the displacement process when computing the recovery but accounts for the mobility ratio when computing the WOR, which results in contradictory formulas for the performance. The Dykstra-Parsons method and its modified version by Johnson[5] use semiempirical correlations based on log-normal distribution of the layers' permeability. Muskat[6] presented analytical expressions for the performance of reservoirs having linear and exponential permeability distributions.

Two methods are available in the literature for estimating the performance of communicating systems with complete crossflow—the method of Warren and Cosgrove[7] and that of Hearn.[8] Warren and Cosgrove's method requires a log-normal permeability distribution. Furthermore, it ignores the problem of ordering of layers for low mobility ratio, which may cause physically meaningless results. The method of Hearn is intended to derive pseudorelative permeability functions for the stratified system to be used in reservoir simulation.

Most of these methods assume that all layers have identical properties except permeability. Also, the time is not related explicitly to the performance. Furthermore, none of these methods considers the variation in injection rate and total pressure drop as the displacement process progresses. Although these points can be treated numerically for a particular case using reservoir simulation methods,[9–13] the objective of this work is to develop analytical expressions for waterflooding performance in idealized linear stratified systems that will consider the previously mentioned points.

Theoretical Analysis

Assumption and Definitions. For both the noncommunicating and communicating systems, these assumptions are made.

1. The system is linear and horizontal, and the flow is incompressible, isothermal, and obeys Darcy's law.

2. The displacement is piston-like, with only residual oil left behind the displacement front and only oil flowing ahead of the displacement front.

3. All layers have the same relative permeability characteristics.

4. Each layer is uniform in thickness, porosity, permeability, and fluid saturations. These properties may, however, vary between the different layers.

5. The displacement is either at constant injection rate or at constant pressure drop.

The following definitions are introduced to get a generalized dimensionless performance.

$$X=\frac{x}{L}, \quad (1)$$

$$\Delta S=1-S_{wi}-S_{or}, \quad (2)$$

$$\gamma=\frac{K_{rw}}{K_{ro}}\frac{\mu_o}{\mu_w}, \quad (3)$$

$$t_D=\frac{\int_0^t Q_t \mathrm{d}Z}{A_t L\bar{\phi}\overline{\Delta S}}, \quad (4)$$

and

$$k_{di}=\frac{k_i}{\phi_i DS_i}\bar{\phi}\overline{\Delta S}. \quad (5)$$

The layers are arranged in decreasing order of k_{di} and the terms are defined in Eqs. 6 through 9.

$$\Delta V_{pj}=\frac{\phi_j \Delta h_j \Delta S_j}{\bar{\phi}\, h_T \overline{\Delta S}}, \quad (6)$$

$$V_{pj}=\sum_{i=1}^{j}\Delta V_{pi}, \quad (7)$$

$$C_j=\sum_{i=1}^{j}k_{di}\Delta V_{pi}, \quad (8)$$

and

$$C_t=C_n=\sum_{i=1}^{n}k_{di}\Delta V_{pi}. \quad (9)$$

Performance of Noncommunicating System. In this case, the layers are assumed completely separated by impermeable thin strata so that no crossflow takes place between the different layers. The performance of such a system is described in this section and details of derivation are given in Appendix A.

The dimensionless time of breakthrough in the first layer, t_{D1}, is given by

$$t_{D1}=\left(\frac{\gamma}{\gamma-1}\right)\left[1-\sum_{i=1}^{n}\Delta V_{pi}\cdot\sqrt{1-\left(\frac{\gamma^2-1}{\gamma^2}\right)\frac{k_{di}}{k_{d1}}}\right]. \quad (10)$$

For $t_D<t_{D1}$, the dimensionless time is related to the fractional distance traveled by the displacement front in the first layer by Eq. 11.

$$t_D=\left(\frac{\gamma}{\gamma-1}\right)\left\{1-\sum_{i=1}^{n}\Delta V_{pi}\cdot\sqrt{1-2\left(\frac{\gamma-1}{\gamma}\right)\frac{k_{di}}{k_{d1}}\left[X_1-\left(\frac{\gamma-1}{2\gamma}\right)X_1^2\right]}\right\}. \quad (11)$$

For the general case where the mobility of the displacing fluid differs from that of the displaced fluid, as displacement progresses, the injection rate changes with time if the total pressure drop is kept constant. On the other hand, if the injection rate is kept constant, the total pressure drop will change with time. In general, the ratio $\Delta p_t/Q_t$ varies according to the relation shown in Eq. 12:

$$\frac{\left(\frac{\Delta P_t}{Q_t}\right)}{\left(\frac{\Delta p_t}{Q_t}\right)_i}=\beta=\frac{C_t}{\sum_{i=1}^{n}\frac{k_{di}\Delta V_{pi}}{\sqrt{1-2\left(\frac{\gamma-1}{\gamma}\right)\frac{k_{di}}{k_{d1}}\left[X_1-\left(\frac{\gamma-1}{2\gamma}\right)X_1^2\right]}}}, \quad (12)$$

where β is the inverse of the injectivity (productivity) ratio.

The actual time, t, is related to the dimensionless time, t_D, as shown in Eq. 13:

$$t=\alpha\frac{A_t\bar{\phi}\overline{\Delta S}}{(Q_t)_i}t_D, \quad (13)$$

where α is the time conversion factor and is given by

$$\alpha=\begin{cases}1 & \text{(for constant } Q_t\text{)}\\ \left[X_1-\frac{\gamma-1}{2\gamma}X_1^2\right]\frac{C_t}{k_{d1}} & \text{(for constant } \Delta p_t\text{).}\end{cases} \quad (14)$$

Before breakthrough in the first layer ($t_D < t_{D1}$), the water cut, W_c, is zero and the fractional oil recovery, N_{pa}, is equal to the dimensionless time, t_D.

$$W_c = 0 \quad \text{.......} (15)$$

and

$$N_{pa} = t_D . \quad \text{.......} (16)$$

After breakthrough in the first layer ($t_D > t_{D1}$), the dimensionless time of breakthrough in the different layers is given by

$$t_{Dj} = \left(\frac{\gamma}{\gamma-1}\right)\left[1 - \left(\frac{\gamma^2+1}{2\gamma}\right)V_{pj} + \left(\frac{\gamma^2-1}{2\gamma}\right)\frac{C_j}{k_{dj}} - \sum_{i=j+1}^{n} \Delta V_{pj}\sqrt{1-\left(\frac{\gamma^2-1}{\gamma^2}\right)\frac{k_{di}}{k_{dj}}}\,\right]. \quad \text{.......} (17)$$

The fractional oil recovery at the time of breakthrough in the jth layer is given by

$$N_{paj} = t_{Dj} - \left(\frac{\gamma+1}{2}\right)\left(\frac{C_j}{k_{dj}} - V_{pj}\right). \quad \text{.......} (18)$$

The water cut, N_c, is given by

$$W_{cj} = \frac{\gamma C_t}{\gamma C_j + \displaystyle\sum_{i=j+1}^{n} \frac{k_{di}\Delta V_{pi}}{\sqrt{1-\left(\frac{\gamma^2-1}{\gamma^2}\right)\frac{k_{di}}{k_{dj}}}}}. \quad \text{.......} (19)$$

The inverse of the injectivity ratio, β, and the time conversion factor, α, are given by Eqs. 20 and 21:

$$\beta_j = \frac{C_t}{\gamma C_j + \displaystyle\sum_{i=j+1}^{n} \frac{k_{di}\Delta V_{pi}}{\sqrt{1-\left(\frac{\gamma^2-1}{\gamma^2}\right)\frac{k_{di}}{k_{dj}}}}}. \quad \text{.......} (20)$$

and

$$\alpha_j = \begin{cases} 1 & \text{(for constant } Q_t) \\ \left(\frac{\gamma+1}{2\gamma}\right)\frac{C_t}{k_{dj}t_{Dj}} & \text{(for constant } \Delta p_t) \end{cases} \quad \text{....} (21)$$

Performance of Communicating Systems With Complete Crossflow. In this case, the layers are assumed completely connected and the crossflow between the different layers is instantaneous such that no vertical pressure drop exists. This implies high vertical flow conductivity because of the large lateral area for crossflow. The performance of this system is described later and details of derivation are given in Appendix B.

The time of breakthrough in the successive layers is given by

$$t_{Dj} = \frac{\Delta V_{pj}}{W_{cj} - W_{cj-1}}, \quad \text{.......} (22)$$

where the water cut, W_c, is given by

$$W_{cj} = \frac{\gamma C_j}{C_t + (\gamma-1)C_j}. \quad \text{.......} (23)$$

The inverse of the injectivity ratio, β, and the time conversion factor, α, are given by

$$\beta_j = \frac{C_t}{C_t + (\gamma-1)C_j} - \delta_j t_{Dj} \quad \text{.......} (24)$$

and

$$\alpha_j = \begin{cases} 1 & \text{(for constant } Q_t) \\ \dfrac{C_t}{C_t + (\gamma-1)C_{j-1}} - ½\delta_{j-1}t_{Dj} + \left(\frac{\gamma-1}{2\gamma}\right)\dfrac{V_{pj-1}}{t_{Dj}} & \text{(for constant } \Delta p_t), \end{cases}$$

$$\text{.......} (25)$$

where

$$\delta_j = \gamma(\gamma-1)C_t^2 \sum_{i=j+1}^{n} \cdot \frac{k^2{}_{di}\Delta V_{pi}}{[C_t + (\gamma-1)C_{i-1}]^2[C_t + (\gamma-1)C_i]^2}. \quad \text{.......} (26)$$

The fractional oil recovery at breakthrough in the jth layer is given by

$$N_{paj} = V_{pj} + \left(\frac{1-W_{cj}}{W_{cj} - W_{cj-1}}\right)\Delta V_{pj}. \quad \text{.......} (27)$$

Before breakthrough in the first layer ($t_D < t_{D1}$), the water cut is zero and the fractional oil is the same as the dimensionless time:

$$W_c = 0 \ \ (\text{for } t_D < t_{D1}) \quad \text{.......} (28)$$

and

$$N_{pa} = t_D \ \ (\text{for } t_D < t_{D1}). \quad \text{.......} (29)$$

The factors β and α are given by

$$\beta=1-\delta_o t_D \quad \text{.......... (30)}$$

and

$$\alpha=\begin{cases} 1 & \text{(for constant } Q_t) \\ 1-\dfrac{\delta_o}{2}t_D & \text{(for constant } \Delta p_t) \end{cases}, \quad \text{.......... (31)}$$

where δ_o is given by general definition of δ_j (Eq. 26) with $j=0$.

Ordering of Communicating System With Favorable Mobility Ratio. It was observed during computations for communicating systems with $\gamma<1$ that when layers are arranged in decreasing reduced permeability, k_d, physically absurd results sometimes may occur. These results included oscillating fractional recoveries and breakthrough times of the different layers and fractional recoveries exceeding unity. This behavior was recognized by Hearn[8] but was ignored by others such as Warren and Cosgrove.[7] Hearn suggested that if such a situation arises, layers should be rearranged in increasing order of t_{Dj} and calculations repeated until the ordering process converges to a system of increasing t_{Dj}. Hearn also recognized that such a procedure may not converge. It is shown later that if such a situation is met where $t_{Dj}>t_{Dj+1}$, the procedure suggested by Hearn will never converge. Hearn also suggested that when the interchange of layers fails to produce a convergent, ordered system, layers of close values of t_{Dj} should be combined to form a single layer. In this work, since we recognized that interchange of layers will fail to produce an ordered system with increasing t_{Dj}, a systematic procedure is used to handle such a situation and is described as follows.

The time of breakthrough in the first layer must be less than unity. For this to be satisfied, we must have

$$\Delta V_{p1}>\frac{1}{(1-\gamma)}\left(\frac{C_t}{k_{d1}}-\gamma\right). \quad \text{.......... (32)}$$

This inequality is checked and, if it is satisfied, we proceed to the next layer. If ΔV_{p1} is not large enough to satisfy Eq. 32, the second layer is combined with the first layer to form a single layer with average properties.

$$\phi_1^*=\frac{\phi_1\Delta V_{p1}+\phi_2\Delta V_{p2}}{\Delta V_{p1}+\Delta V_{p2}}, \quad \text{.......... (33)}$$

$$\Delta S_1^*=\frac{\phi_1\Delta S_1\Delta V_{p1}+\phi_2\Delta S_2\Delta V_{p2}}{\phi_1\Delta V_{p1}+\phi_2\Delta V_{p2}}, \quad \text{.......... (34)}$$

$$k_1^*=\frac{k_1\Delta V_{p1}+k_2\Delta V_{p2}}{\Delta V_{p1}+\Delta V_{p2}}, \quad \text{.......... (35)}$$

$$\Delta V_{p1}^*=\Delta V_{p1}+\Delta V_{p2}, \quad \text{.......... (36)}$$

and

$$k_{d1}^*=\frac{k_1^*}{\phi_1^*\Delta S_1^*}\,\overline{\phi\Delta S}. \quad \text{.......... (37)}$$

The inequality in Eq. 32 is checked again with the new values of ΔV_{p1}^* and k_{d1}^*. Additional layers may be added and the procedure repeated until this condition is satisfied.

For successive layers and after t_{Dj} is calculated and the condition $t_{Dj}>t_{Dj-1}$ is satisfied, the condition for the time of breakthrough in the next layer t_{Dj+1} to be greater than t_{Dj} as obtained from Eqs. 22 and 23 is

$$\frac{k_{dj}-k_{dj+1}}{k_{dj}(k_{dj}\Delta V_{pj}+k_{dj+1}\Delta V_{pj+1})}>\frac{(1-\gamma)}{C_t-(1-\gamma)C_{j-1}}. \quad \text{.......... (38)}$$

If this condition is not satisfied, the $j+2$ layer is combined with the $j+1$ layer to form the new $j+1$ layer, with average properties given by equations similar to Eqs. 33 through 37. The inequality in Eq. 38 is checked again and more layers may be added until it is satisfied.

Finally, if the last layer gives a value of t_{DN} less than t_{Dn-1}, then the last layer is combined with the layer directly above it.

Inspection of Eq. 38 shows that if the inequality is not satisfied, then the interchange of layers j and $j+1$ yield the following situation.

$$k_{dj}^*=k_{dj+1}, \quad \text{.......... (39)}$$

$$\Delta V_{pj}^*=\Delta V_{pj+1}, \quad \text{.......... (40)}$$

$$k_{dj+1}^*=k_{dj}, \quad \text{.......... (41)}$$

and

$$\Delta V_{pj+1}^*=\Delta V_{pj}. \quad \text{.......... (42)}$$

The left side of the inequality shown in Eq. 38 becomes

$$\frac{k_{dj}^*-k_{dj+1}^*}{k_{dj}^*\;(k_{dj}^*\Delta V_{pj}^*+k_{dj+1}^*\Delta V_{pj+1}^*)}$$

$$=\frac{k_{dj+1}-k_{dj}}{k_{dj+1}(k_{dj+1}\Delta V_{pj+1}+k_{dj}\Delta V_{pj})}, \quad \text{.......... (43)}$$

which is negative since $k_{dj}>k_{dj+1}$ according to the first arrangement of layers. Therefore, the condition given by the inequality in Eq. 38 will not be satisfied by interchange of layers as was stated previously. Only combination of one or more layers with the $j+1$ layer may satisfy the inequality in Eq. 38.

Displacement With Unit Mobility Ratio. For the case where $\gamma=1$, both models should predict identical performance. However, the equations derived for the noncom-

TABLE 1—PERMEABILITY AND POROSITY DISTRIBUTION*

Layer	Δh (ft)	k (md)	ϕ $\sigma_\phi=0$	$\sigma_\phi=0.02$	$\sigma_\phi=0.05$
1	1	1,000.0	0.2	0.2353	0.2879
2	1	795.0	0.2	0.2293	0.2733
3	2	587.5	0.2	0.2216	0.2540
4	2	432.0	0.2	0.2137	0.2344
5	2	348.5	0.2	0.2083	0.2207
6	2	280.5	0.2	0.2027	0.2068
7	2	230.0	0.2	0.1977	0.1942
8	2	188.0	0.2	0.1925	0.1813
9	2	149.0	0.2	0.1870	0.1670
10	2	111.0	0.2	0.1800	0.1500
11	1	81.0	0.2	0.1700	0.1300
12	1	35.0	0.2	0.1500	0.0700

*$\sigma_k=0.784$, $k_m=252$ md, $\phi_m=0.2$, and $\Delta S=0.6$.

municating case cannot be used since they include terms that become indeterminate as γ approaches unity. The performance of the two models also should be identical to that predicted by Stiles, except that Stiles' model does not account for variation in porosity and fluid saturation between the different layers. The performance for unit mobility ratio thus is deduced from the case of communicating system with $\gamma=1$. In this case, the performance equations reduce to

$$t_{Dj}=\frac{C_t}{k_{dj}}, \quad \text{(44)}$$

$$W_{cj}=\frac{C_j}{C_t}, \quad \text{(45)}$$

$$N_{paj}=V_{pj}+\frac{C_t-C_j}{k_{dj}}, \quad \text{(46)}$$

and

$$\alpha_j=\beta_j=1. \quad \text{(47)}$$

Results and Discussions

The developed models are applied to the example problem reported by Warren and Cosgrove.[7] Results were obtained for both noncommunicating and communicating systems at mobility ratios of 0.5, 1, 2, and 5. The effect of variable porosity also was investigated by considering systems with normal porosity distributions at different mobility ratios for both communicating and noncommunicating cases. The data for the different cases studied are given in Table 1. The results obtained are shown in Figs. 1 through 6. More details can be found in Ref. 14.

Effect of Mobility Ratio. The performance of the stratified system is shown in Fig. 1 for the noncommunicating case and in Fig. 2 for the communicating system. In these figures, the water cut W_c is plotted vs. the fractional oil recovery N_{pa}. In Fig. 3, N_{pa} is plotted vs. the dimensionless time t_D. The performance was estimated at mobility ratios of 0.5, 1, 2, and 5. As expected, the fractional oil recovery at the same value of W_c increases as the mobility ratio is decreased. Note, however, that the effect of mobility ratio on oil recovery is more pronounced for the case of communicating systems with crossflow than for the case of noncommunicating systems without crossflow.

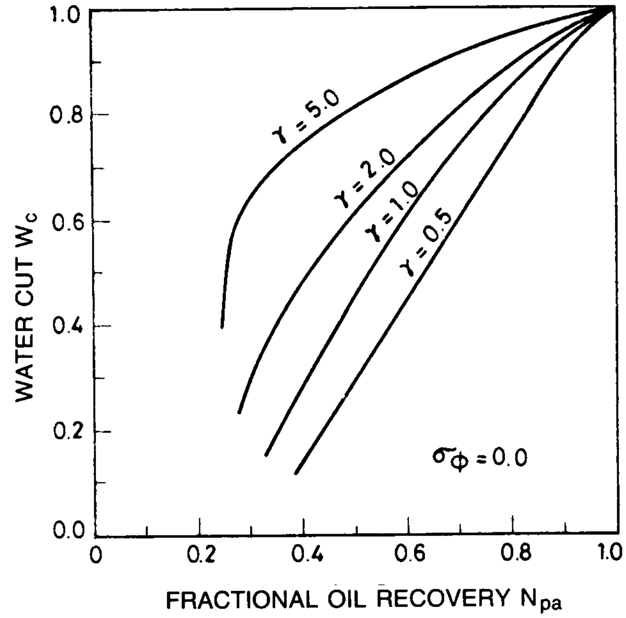

Fig. 1—Performance of noncommunicating system.

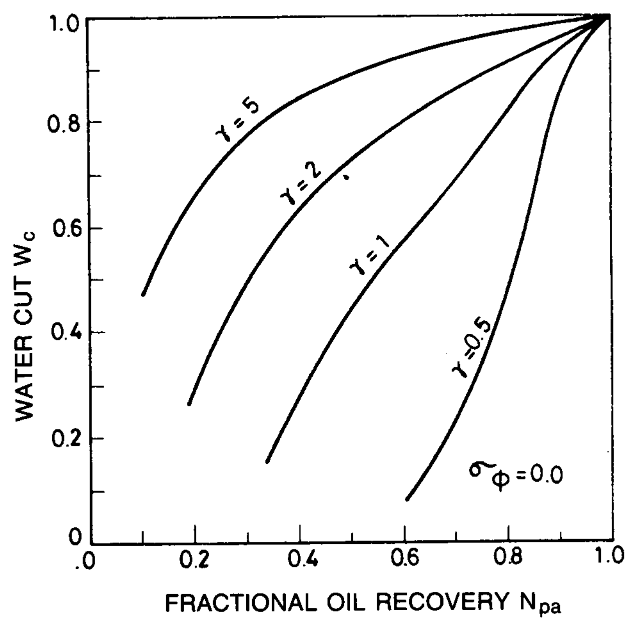

Fig. 2—Performance of communicating system.

Effect of Crossflow. It can be seen from Figs. 1 through 3 that at favorable mobility ratios ($\gamma<1$), the fractional oil recovery is higher for the communicating system with crossflow than for the noncommunicating system without crossflow. At unfavorable mobility ratios ($\gamma>1$), the opposite is true. For unit mobility ratio ($\gamma=1$), the performance of the two systems is identical.

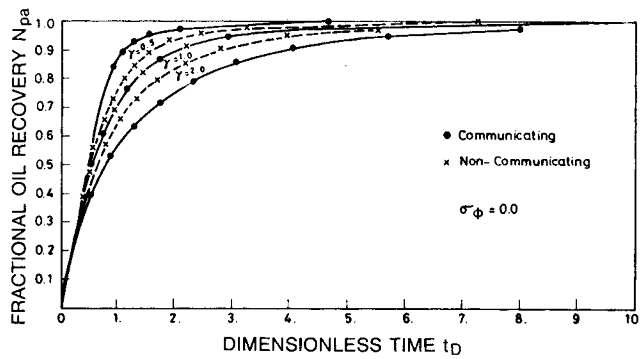

Fig. 3—Effect of crossflow on performance.

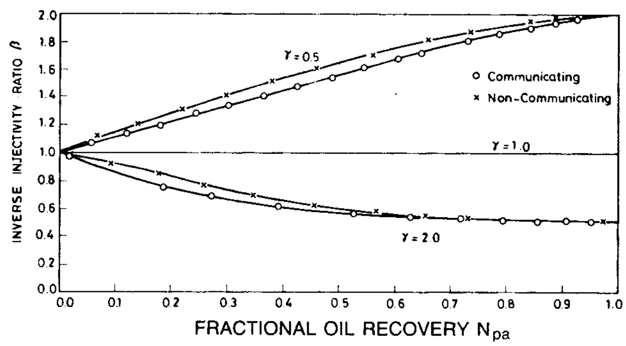

Fig. 4—Variation of injectivity ratio.

Variation of the Injectivity Ratio. Fig. 4 shows the inverse of the injectivity ratio, β, plotted vs. fractional oil recovery at mobility ratios of 0.5, 1, and 2 for both cases of noncommunicating and communicating systems. At unit mobility ratio, β remains constant at a value of 1.0 for both cases. For other mobility ratios, the ratio β varies from 1.0 at $N_{pa}=0$ to a value of $1/\gamma$ at $N_{pa}=1.0$. This implies that for favorable mobility ratios ($\gamma<1$) the ratio β increases continuously as the displacement progresses. In this case, the injection rate decreases or the total pressure drop increases as more of the less mobile displacing fluid enters the system. For unfavorable mobility ratios ($\gamma>1$), the opposite is true. It can be seen in Fig. 4 that the value of β for communicating systems is always less than that for the noncommunicating systems. This indicates that crossflow tends to increase the injection rate and/or decrease the pressure drop in the system.

Effect of Variable Porosity and Saturation. The effect of variable porosity is investigated by considering stratified systems in which the porosity is normally distributed and depends on permeability according to the following relation.

$$\phi=\bar{\phi}+\frac{\sigma_\phi}{\sigma_k}\ln\left(\frac{k}{k_m}\right). \quad \text{(48)}$$

The performance was computed for values of σ_ϕ of 0.02 and 0.05 at mobility ratios of 0.5, 1.0, 2.0, and 5.0 for both noncommunicating and communicating cases. The results are shown in Figs. 5 and 6, together with those for the constant-porosity case ($\sigma_\phi=0.0$). It is seen from these figures that oil recovery is always higher for the variable-porosity case than for the constant-porosity case and that recovery increases as σ_ϕ increases. Since porosity is assumed to be directly proportional to permeability, it follows that layers with high permeabilities also will have higher porosities. Since the rate of advance of the displacement front is proportional to $k/\phi\Delta S$, it follows that the effect of variable porosity is to retard the rate of advance in the more permeable layers and accelerate it in the less permeable layers. This effect tends to narrow the length of the displacement front and results in higher oil recovery and lower water cut. This effect is noticed to be almost independent of mobility ratio for noncommunicating systems.

For systems with crossflow, the effect of porosity variation is found to increase at favorable mobility ratios and to decrease at unfavorable mobility ratios.

Since the connate water saturation, S_{wi}, and the residual oil saturation decrease as the absolute permeability increases, the value of $\Delta S=1-S_{wi}-S_{or}$ is expected to increase as k increases. The effect of variable

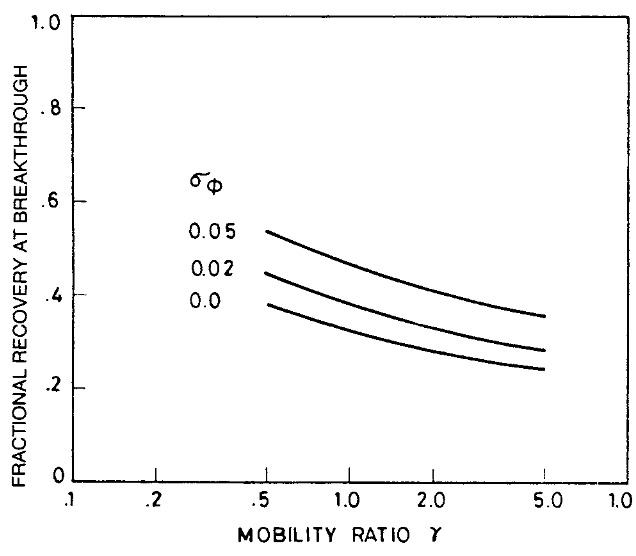

Fig. 5—Effect of variable porosity—noncommunicating case.

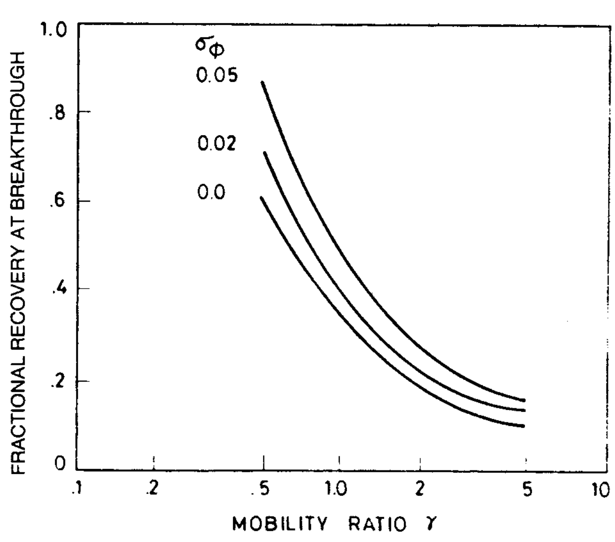

Fig. 6—Effect of variable porosity—communicating case.

fluid saturation thus is expected to be similar to the effect of variable porosity, since the rate of advance of the displacement front is proportional to $k/\phi\Delta S$. The two factors, porosity and fluid saturation, could be combined into one variable, $\phi\Delta S$.

Conclusions

1. A mathematical model was developed to predict the waterflooding performance in a linear stratified reservoir with and without crossflow. The model assumes no particular permeability distribution and accounts for variation in other rock properties, such as porosity and fluid saturations. A procedure for ordering and combination of layers when necessary is outlined.
2. The effect of crossflow between layers is found to increase the oil recovery at favorable mobility ratios and to decrease it at unfavorable mobility ratios. Crossflow also tends to make the influence of mobility ratio in flooding performance more pronounced.
3. The injectivity ratio ($1/\beta$) changes as the displacement progresses. It increases for unfavorable mobility ratios and decreases for favorable mobility ratios. The effect of crossflow is to increase the injection rate and/or decrease the pressure drop at the same mobility ratio.
4. Variation in porosity and fluid saturation between the different layers increases oil recovery over that for constant $\phi\Delta S$. This effect is almost independent of mobility ratio for noncommunicating systems and increases as the mobility ratio decreases for communicating systems.

Nomenclature

A = area, sq ft [m^2]
C = formation capacity, md-ft [m^3]
h_t = total thickness, ft [m]
Δh = thickness of layer, ft [m]
k = absolute permeability, md
k_d = reduced permeability, md
k_m = mean permeability, md
k_{ro}^o = oil relative permeability at initial water saturation
k_{rw}^o = water relative permeability at residual oil saturation
L = length of stratified system, ft [m]
n = number of layers
N_{pa} = fractional oil recovery
p = pressure, psi [kPa]
Δp = pressure drop, psi [kPa]
Q = flow rate, bbl/D [m^3/h]
S_{or} = residual oil saturation
S_{wi} = initial water saturation
ΔS = saturation change at the front
$\overline{\Delta S}$ = average saturation change
t = time, hours
t_D = dimensionless time
ΔV_p = hydrocarbon pore volume
W = water
x = distance traveled by displacement front, ft [m]
X = dimensionless distance
Z = dummy variable of integration
α = time conversion factor
β = inverse injectivity ratio
γ = mobility ratio
Δ = difference operator
μ = viscosity, cp [Pa·s]
σ = standard deviation
ϕ = porosity
$\bar{\phi}$ = average porosity

Subscripts

c = cut
i = initial
k = permeability
o = oil
t = total
w = water
ϕ = porosity

Superscripts

$^-$ = average

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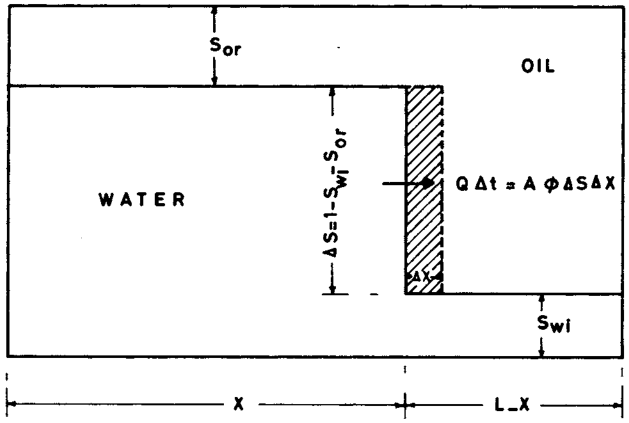

Fig. A-1—Saturation distribution and displacement front.

APPENDIX A

Derivation of the Mathematical Model for Noncommunicating Systems

Since no crossflow is allowed between layers, the flow rate through any layer i is independent of position and is given by (see Fig. A-1)

$$Q_i=\frac{k_i k_{rw}^o}{\mu_w}A_i\left(\frac{\Delta p_1}{x_1}\right)_i$$

$$=k_i\frac{k_{ro}^o}{\mu_o}A_i\left(\frac{\Delta p_2}{L-x_1}\right)_i . \qquad \text{(A-1)}$$

The total pressure drop, Δp_t, which is the same for all layers, is obtained by adding Δp_1 and Δp_2 from Eq. A-1 and rearranging:

$$\Delta p_t=\frac{Q_i L\mu_o}{k_i A_i k_{ro}^o}\left[1-\left(\frac{\gamma-1}{\gamma}\right)X_i\right] \qquad \text{(A-2)}$$

and

$$Q_i=\frac{k_i A_i k_{ro}^o}{\mu_o L}\frac{p_t}{\left[1-\left(\frac{\gamma-1}{\gamma}\right)X_i\right]} . \qquad \text{(A-3)}$$

The total flow rate Q_t is obtained by adding Q_i's in all the layers:

$$Q_t=\frac{k_{ro}^o\,\Delta p_t}{\mu_o L}\sum_{i=1}^{n}\frac{k_i A_i}{1-\left(\frac{\gamma-1}{\gamma}\right)X_i} . \qquad \text{(A-4)}$$

Initially, $X_i=0$ for all layers and

$$(Q_t)_i=\frac{k_{ro}^o}{\mu_o L}(\Delta p_t)_i\sum_{i=1}^{n}k_i A_i . \qquad \text{(A-5)}$$

From Eqs. A-4 and A-5, by using dimensionless definitions,

$$\beta=\left(\frac{\Delta p_t}{Q_t}\right)\Big/\left(\frac{\Delta p_t}{Q_t}\right)_i=\frac{C_t}{\sum_{i=1}^{n}\frac{k_{di}\Delta V_{pi}}{1-\left(\frac{\gamma-1}{\gamma}\right)X_i}} . \qquad \text{(A-6)}$$

The rate of advance of the displacement front is given by

$$\frac{dx_i}{dt}=\frac{Q_i}{A_i\phi_i\Delta S_i}, \qquad \text{(A-7)}$$

and, in dimensionless form,

$$\frac{dX_i}{dt_D}=\frac{Q_i}{Q_t\Delta V_{pi}} . \qquad \text{(A-8)}$$

Substituting for the values of Q_i and Q_t from Eqs. A-3 and A-4, respectively, we get

$$\frac{dX_i}{dt_D}=\frac{k_{di}}{\left[1-\left(\frac{\gamma-1}{\gamma}\right)X_i\right]\sum_{i=1}^{n}\frac{k_{di}\Delta V_{pi}}{1-\left(\frac{\gamma-1}{\gamma}\right)X_i}} . \qquad \text{(A-9)}$$

Eq. A-9 can be applied for two different layers, i and j, giving

$$\frac{dX_i}{dX_j}=\frac{k_{di}}{k_{dj}}\frac{\left[1-\left(\frac{\gamma-1}{\gamma}\right)X_j\right]}{\left[1-\left(\frac{\gamma-1}{\gamma}\right)X_i\right]} . \qquad \text{(A-10)}$$

Eq. A-10 is integrated and solved for X_i in terms of X_j. The result is

$$X_i=\left(\frac{\gamma}{\gamma-1}\right)\cdot\left\{1-\sqrt{1-2\left(\frac{\gamma-1}{\gamma}\right)\frac{k_{di}}{k_{dj}}\left[X_j-\left(\frac{\gamma-1}{2\gamma}\right)X_j^2\right]}\right\} . \qquad \text{(A-11)}$$

At water breakthrough in the jth layer, $X_j=1$, and for $i>j$, Eq. A-11 becomes

$$X_i=\left(\frac{\gamma}{\gamma-1}\right)\left[1-\sqrt{1-\left(\frac{\gamma^2-1}{\gamma^2}\right)\frac{k_{di}}{k_{dj}}}\right] . \qquad \text{(A-12)}$$

Substituting Eq. A-12 into Eq. A-6 and noting that

$X_i=1$ for $i \le j$,

we get

$$\beta_j=\frac{C_t}{\gamma C_j+\sum_{i=j+1}^{n}\frac{k_{di}\Delta V_{pi}}{\sqrt{1-\left(\frac{\gamma^2-1}{\gamma^2}\right)\frac{k_{di}}{k_{dj}}}}}. \quad \text{(A-13)}$$

Before breakthrough in the first layer, the ratio β is obtained by substituting Eq. A-11 into Eq. A-6, expressing all X_i's in terms of any X_j. Using the fractional distance moved by the displacement front in the first layer, X_1, we get

$$\beta=\frac{C_t}{\sum_{i=1}^{n}\frac{k_{di}\Delta V_{pi}}{\sqrt{1-2\left(\frac{\gamma-1}{\gamma}\right)\frac{k_{di}}{k_{d1}}\left[X_1-\left(\frac{\gamma-1}{2\gamma}\right)X_1^2\right]}}} \quad \text{(A-14)}$$

The fractional oil recovery, N_{pa}, expressed in terms of recoverable oil is given by

$$N_{pa}=\sum_{i=1}^{n}\Delta V_{pi}X_i. \quad \text{(A-15)}$$

Substituting for X_i in terms of X_1 by using Eq. A-11, we get the expression for N_{pa} before breakthrough in the first layer, which is the same as the dimensionless time, t_D:

$$N_{pa}=t_D=\left(\frac{\gamma}{\gamma-1}\right)\left\{1-\sum_{i=1}^{n}\cdot\Delta V_{pi}\sqrt{1-2\left(\frac{\gamma-1}{\gamma}\right)\frac{k_{di}}{k_{d1}}\left[X_1-\left(\frac{\gamma-1}{\gamma}\right)X_1^2\right]}\right\}. \quad \text{(A-16)}$$

At the time of breakthrough in the jth layer, $X_i=1$ for $i \le j$, and X_i is given by Eq. A-12 for $i>j$. Substituting into Eq. A-15, we get

$$N_{pa}=V_{pj}+\left(\frac{\gamma}{\gamma-1}\right)\cdot\left[(1-V_{pj})-\sum_{i=j+1}^{n}\Delta V_{pi}\sqrt{1-\left(\frac{\gamma^2-1}{\gamma^2}\right)\frac{k_{di}}{k_{dj}}}\right]$$

$$=\left(\frac{\gamma}{\gamma-1}\right)\left[1-\frac{V_{pj}}{\gamma}-\sum_{i=j+1}^{n}\Delta V_{pi}\cdot\sqrt{1-\left(\frac{\gamma^2-1}{\gamma^2}\right)\frac{k_{di}}{k_{dj}}}\right]. \quad \text{(A-17)}$$

Eq. A-16 also could be obtained by integrating Eq. A-9 after substituting for X_j in terms of X_1 by using Eq. A-11. At breakthrough in the jth layer, Eq. A-9 for the $j+1$ layer after using Eq. A-12 takes the form

$$\frac{dX_{j+1}}{dt_D}=k_{dj+1}\div\left[1-\left(\frac{\gamma-1}{\gamma}\right)X_{j+1}\right]\left\{\gamma C_j+\sum_{i=j+1}^{n}\cdot\frac{k_{di}\Delta V_{pi}}{\sqrt{1-2\left(\frac{\gamma-1}{\gamma}\right)\frac{k_{di}}{k_{dj+1}}\left[X_{j+1}-\left(\frac{\gamma-1}{2\gamma}\right)X_j{}^2{}_{+1}\right]}}\right\}. \quad \text{(A-18)}$$

Integrating this equation between X_{j+1} at breakthrough in the jth layer and $X_{j+1}=1$, we get the dimensionless time between breakthrough in the jth and $j+1$ layers.

$$\Delta t_{Dj+1}=\left(\frac{\gamma+1}{2}\right)C_j\left(\frac{1}{k_{dj+1}}-\frac{1}{k_{dj}}\right)+\left(\frac{\gamma}{\gamma-1}\right)\cdot\sum_{i=j+1}^{n}\Delta V_{pi}\sqrt{1-\left(\frac{\gamma^2-1}{\gamma^2}\right)\frac{k_{di}}{k_{dj}}}-\left(\frac{\gamma}{\gamma-1}\right)\sum_{i=j+1}^{n}\cdot\Delta V_{pi}\sqrt{1-\left(\frac{\gamma^2-1}{\gamma^2}\right)\frac{k_{di}}{k_{dj+1}}}. \quad \text{(A-19)}$$

Since $t_{Dj}=\sum_{i=1}^{j}\Delta t_{Di}$, and noting that for $j=0$, $k_{do}=\infty$ and $C_o=0$, we get

$$t_{Dj}=\left(\frac{\gamma}{\gamma-1}\right)\left[1-\left(\frac{\gamma^2+1}{2\gamma}\right)V_{pj}+\left(\frac{\gamma^2-1}{2\gamma}\right)\frac{C_j}{k_{dj}}\right.$$
$$\left.-\sum_{i=j+1}^{n}\Delta V_{pi}\sqrt{1-\left(\frac{\gamma^2-1}{\gamma^2}\right)\frac{k_{di}}{k_{dj}}}\right]. \quad \text{(A-20)}$$

The water cut at time of breakthrough in the jth layer, W_{cj}, is given by

$$W_{cj}=\sum_{i=1}^{j} Q_i/Q_t. \quad \text{(A-21)}$$

Substituting for Q_i and Q_t from Eqs. A-3 and A-4, respectively, and noting that $X_i=1$ for $i\leq j$ and X_i is given by Eq. A-12 for $i>j$, we get

$$W_{cj}=\frac{\gamma C_j}{\gamma C_j+\sum_{i=j+1}^{n}\frac{k_{di}\Delta V_{pi}}{\sqrt{1-\left(\frac{\gamma^2-1}{\gamma^2}\right)\frac{k_{di}}{k_{dj}}}}} \quad \text{(A-22)}$$

To convert the dimensionless time, t_D, into real time, t, for constant injection rate, one uses

$$t=\frac{A_t\bar{\phi}\overline{\Delta S}}{Q_t}t_D. \quad \text{(A-23)}$$

For the case of constant pressure drop, Q_t is not constant. Substituting Eqs. A-3 and A-5 into Eq. A-7 we get

$$\frac{dX_i}{dt}=\frac{(Q_t)_i k_{di}}{C_t A_t\bar{\phi}L\overline{\Delta S}\left[1-\left(\frac{\gamma-1}{\gamma}\right)X_i\right]}. \quad \text{(A-24)}$$

This equation is integrated to give

$$t=\frac{C_t}{k_{di}}\frac{A_t\bar{\phi}\overline{\Delta S}L}{(Q_t)_i}\left[X_i-\left(\frac{\gamma-1}{2\gamma}\right)X_i^2\right]. \quad \text{(A-25)}$$

Introducing the time conversion factor such that

$$t=\alpha\frac{A_t\bar{\phi}\overline{\Delta S}L}{(Q_t)_i}t_D, \quad \text{(A-26)}$$

and comparing Eqs. A-25 and A-26, we get

$$\alpha=\frac{C_t}{t_D k_{di}}\left[X_i-\left(\frac{\gamma-1}{2\gamma}\right)X_i^2\right]. \quad \text{(A-27)}$$

Note that $[X_i-(\gamma-1)/(2\gamma)X_i^2]/k_{di}$ is the same for all layers before their time of breakthrough. At breakthrough in the jth layer, $X_j=1$ and

$$\alpha_j=\left(\frac{\gamma+1}{2\gamma}\right)\frac{C_t}{t_{Dj}k_{dj}}. \quad \text{(A-28)}$$

It should be noted that the model could be used for continuous permeability distributions by replacing the summation terms by their equivalent integrals. In general,

$$\sum_{i=1}^{j} f_i\Delta V_{pi}=\int_0^{V_{pj}} f\mathrm{d}V_p, \quad \text{(A-29)}$$

$$\sum_{i=j+1}^{n} f_i\Delta V_{pi}=\int_{V_{pj}}^{1} f\mathrm{d}V_p, \quad \text{(A-30)}$$

and

$$\sum_{i=1}^{n} f_i\Delta V_{pi}=\int_0^{1} f\mathrm{d}V_p, \quad \text{(A-31)}$$

where f_i is any function of V_{pj}, and

$$V_{pj}=\frac{\int_0^{h_j}\phi\Delta S\mathrm{d}h}{\bar{\phi}\overline{\Delta S}h_t}. \quad \text{(A-32)}$$

APPENDIX B

Derivation of the Mathematical Model for Communicating Systems

The communication between the different layers is assumed complete and crossflow is instantaneous such that there is no pressure gradient in the vertical direction. In Zone j, as shown in Fig. B-1, layers 1, 2, 3 . . . j are flowing water while the rest of the layers are flowing oil:

$$Q_{wi}=\frac{k_i k_{rw}^o}{\mu_o}A_i\frac{\partial p}{\partial X},\quad (1\leq i\leq j), \quad \text{(B-1)}$$

and

$$Q_{0i}=\frac{k_i k_{ro}^o}{\mu_o}A_i\frac{\partial p}{\partial X},\quad (j+1\leq i\leq n). \quad \text{(B-2)}$$

From Eqs. B-1 and B-2:

$$W_{cj}=\frac{\sum_{i=1}^{j}Q_{wi}}{\sum_{i=1}^{j}Q_{wi}+\sum_{i=j+1}^{n}Q_{0i}}=\frac{\gamma C_j}{\gamma C_j+(C_t-C_j)}. \quad \text{(B-3)}$$

The rate of advance of water in the jth layer is obtained by making material balance around X_j:

$$\frac{dx_j}{dt}=\frac{Q_t \Delta W_{cj}}{A_i \phi_i \Delta S_w}. \quad \text{(B-4)}$$

In dimensionless form,

$$\frac{dX_j}{dt_D}=\frac{\Delta W_{cj}}{\Delta V_{pj}} \quad \text{(B-5)}$$

and

$$X_j=\frac{\Delta W_{cj}}{\Delta V_{pj}} t_D, \quad \text{(B-6)}$$

where $W_{cj}=W_{cj}-W_{cj-1}$ and $W_{co}=0$, $W_{cn}=1$. Breakthrough in the jth layer occurs when $X_j=1$, hence

$$t_{Dj}=\frac{1}{\dfrac{\Delta W_{cj}}{\Delta V_{pj}}}=\frac{\Delta V_{pj}}{\Delta W_{cj}}. \quad \text{(B-7)}$$

The fractional oil recovery at time of breakthrough in the jth layer is given by

$$N_{paj}=V_{pj}+\sum_{i=j+1}^{n} \Delta V_{pi} X_i$$

$$=V_{pj}+\sum_{j+1}^{n} \Delta V_{pi} \frac{\Delta W_{ci}}{\Delta V_{pi}} t_{Dj}$$

$$=V_{pj}+(1-W_{cj})t_{Dj}. \quad \text{(B-8)}$$

Using Eq. B-7, Eq. B-8 can be written in this form:

$$N_{paj}=V_{pj}+\frac{1-W_{cj}}{\dfrac{\Delta W_{cj}}{\Delta V_{pj}}}. \quad \text{(B-9)}$$

Note that Eqs. B-5, B-7, and B-9 are similar to those for the Buckley-Leverett displacement, with V_{pj} replaced by S_w and the difference operator, Δ, replaced by the differential operator, d.

From Eqs. B-1 and B-2,

$$Q_t=\left(\frac{k_{rw}^{o}}{\mu_w}\sum_{i=1}^{j} k_i A_i+\frac{k_{ro}^{o}}{\mu_o}\sum_{i=j+1}^{n} k_i A_i\right)\frac{\partial p}{\partial x}$$

$$=\frac{k_{ro}^{o}}{\mu_o}\left[\gamma C_t+(C_t-C_j)\right]\frac{A_t}{L}\frac{\partial p}{\partial X}. \quad \text{(B-10)}$$

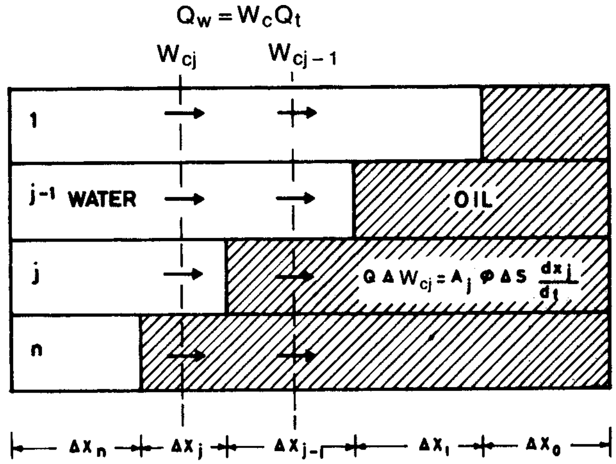

Fig. B-1—Stratified layers with crossflow.

Integrating over the jth zone, we get

$$\Delta p_j=\frac{Q_t \mu_o L}{k_{ro}^{o} A_t}\frac{\Delta X_j}{\gamma C_j+(C_t-C_j)}, \quad \text{(B-11)}$$

where $\Delta X_j=X_j-X_{j+1}$ and $X_o=1$, $X_{n+1}=0$.

The total pressure drop is given by

$$\Delta p_t=\frac{Q_t \mu_o L}{k_{ro}^{o} A_t}\sum_{j=0}^{n}\frac{\Delta X_j}{\gamma C_j+(C_t-C_j)}. \quad \text{(B-12)}$$

For the case of constant pressure drop, Q_t is not constant. Substituting Eqs. A-3 and A-5 into Eq. A-7 we get

$$(\Delta p_t)_i=\frac{(Q_t)_i \mu_o L}{k_{ro}^{o} A_t C_t}. \quad \text{(B-13)}$$

Dividing Eq. B-10 by Eq. B-11 and rearranging:

$$\beta=\frac{\left(\dfrac{\Delta p_t}{Q_t}\right)}{\left(\dfrac{\Delta p_t}{Q_t}\right)_i}=C_t\sum_{j=0}^{n}\frac{\Delta X_j}{\gamma C_j+(C_t-C_j)}. \quad \text{(B-14)}$$

At time of breakthrough in the jth layer, $X_j=0$ and

$$\beta_j=C_t\sum_{i=j}^{n}\frac{X_i-X_{i+1}}{\gamma C_i+C_t-C_i}. \quad \text{(B-15)}$$

Using Eqs. B-6 and B-3, Eq. B-13 may be written in this form:

$$\beta_j=\frac{C_t}{C_t+(\gamma-1)C_j}-\gamma(\gamma-1)C_t t_{Dj}\sum_{i=j+1}^{n}$$

$$\cdot\frac{k_i^2 \Delta V_{pi}}{[C_t+(\gamma-1)C_{i-1}]^2[C_t+(\gamma-1)C_i]^2}. \quad \text{(B-16)}$$

Before breakthrough in the first layer,

$$\beta=1-\gamma(\gamma-1)C_t t_D \sum_{j=1}^{n}$$

$$\cdot\frac{k_{di}^2\,\Delta V_{pi}}{[C_t+(\gamma-1)C_{i-1}]^2[C_t+(\gamma-1)C_i]^2}, \quad \text{(B-17)}$$

Eqs. B-16 and B-17 may be written in the general form

$$\beta_j=\frac{C_t}{C_t+(\gamma-1)C_j}-\delta_j t_D, \quad \text{(B-18)}$$

where

$$\delta_j=\gamma(\gamma-1)C_t \sum_{i=j+1}^{n}$$

$$\cdot\frac{k_{di}^2\,\Delta V_{pi}}{[C_t+(\gamma-1)C_{i-1}]^2[C_t+(\gamma-1)C_i]^2}. \quad \text{(B-19)}$$

For constant Δp_t, Eq. B-16 can be written as

$$\frac{1}{Q_t}=\frac{1}{(Q_t)_{\text{in}}}\left[\frac{C_t}{C_t+(\gamma-1)C_j}-\delta_j t_D\right]. \quad \text{(B-20)}$$

Differentiating with respect to t, we get

$$\frac{d}{dt}\left(\frac{1}{Q_t}\right)=-\frac{1}{(Q_t)_i}\delta_j\frac{Q_t}{A_t\phi L\overline{\Delta S}_w} \quad \text{(B-21)}$$

and

$$\frac{d}{dt}\left(\frac{1}{Q_t{}^2}\right)=-\frac{2\delta_j}{Q_{t_i}A_t\phi L\overline{\Delta S}_w}. \quad \text{(B-22)}$$

Integrating between t_j and t,

$$\frac{1}{Q_t{}^2}=\frac{1}{Q_t{}^2}-\frac{2\delta_j}{Q_{t_i}A_t\phi\overline{\Delta S}_w L}(t-t_j). \quad \text{(B-23)}$$

It is seen that $1/(Q_t{}^2)$ varies linearly between the times of breakthrough of the successive layers.

Using Eq. B-20, we get

$$\frac{1}{(Q_t{}^2)_i}\left[\frac{C_t}{C_t+(\gamma-1)C_j}-\delta_j t_D\right]^2-\frac{1}{(Q_t{}^2)_i}$$

$$\cdot\left[\frac{C_t}{C_t+(\gamma-1)C_j}-\delta_j t_{Dj}\right]^2=\frac{-2\delta_j}{Q_{t_i}A_t\phi\overline{\Delta S}_w L}(t-t_j).$$

$$\text{(B-24)}$$

At $t=t_{j+1}$, $t_D=t_{Dj+1}$ and Eq. B-24 becomes

$$\left[\frac{2C_t}{C_t+(\gamma-1)C_j}-\delta_j(t_{Dj}+t_{Dj+1})\right](t_{Dj+1}-t_{Dj})$$

$$=\frac{2(Q_t)_i(t_{j+1}-t_j)}{A_t\phi\overline{\Delta S}_w L}$$

or

$$\frac{\Delta t_{j+1}(Q_t)_i}{A_t\phi L\overline{\Delta S}_w}=\Delta t_{Dj+1}\left[\frac{C_t}{C_t+(\gamma-1)C_j}\right.$$

$$\left.-\frac{\delta_j}{2}(t_{Dj}+t_{Dj+1})\right]. \quad \text{(B-25)}$$

Since $t_j=\sum_{i=1}^{j}\Delta t_i$, performing the summation we get

$$\frac{t_j(Q_t)_i}{A_t\phi L\overline{\Delta S}_w t_{Dj}}=\alpha_j=\frac{C_t}{C_t+(\gamma-1)C_{j-1}}$$

$$+\frac{(\gamma-1)}{2\gamma}\frac{V_{pj-1}}{t_{Dj}}-½\delta_{j-1}t_{Dj}. \quad \text{(B-26)}$$

Before breakthrough in the first layer,

$$\alpha=1-½\delta_o t_D. \quad \text{(B-27)}$$

For systems with continuous permeability distribution, the same equations of the model could be used if the difference operator, Δ, is replaced by the differential operator, d, and the summation terms are replaced by their equivalent integrals. In particular,

$$\frac{\Delta W_{cj}}{\Delta V_{pj}}=\left(\frac{dW_c}{dV_p}\right)_{V_{pj}}. \quad \text{(B-28)}$$

and

$$\sum_{i=j+1}^{n}\frac{k_{di}{}^2\Delta V_{pi}}{[C_t+(\gamma-1)C_{i-1}]^2[C_t+(\gamma-1)C_i]^2}$$

$$=\int_{V_{pj}}^{1}\frac{k_d{}^2 Z dV_p}{[C_t+(\gamma-1)C]^4}. \quad \text{(B-29)}$$

SI Metric Conversion Factor

ft × 3.048* E−01 = m

*Conversion factor is exact.

SPEJ

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Discussion of the Effect of Crossflow on Waterflooding of Stratified Reservoirs

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S.T. Wang, SPE, Petroleum Recovery Inst.

This Discussion addresses El-Khatib's paper that appeared in the April 1985 *Soc. Pet. Eng. J.* (Pages 291–302).

Stratification built into a three-dimensional model, as reflected in the number of layers and their associated vertical transmissibilities, can have a crucial influence on the simulated recovery. It is commendable for El-Khatib to attempt to illustrate the influence of the variables involved by means of a mathematical model. However, judging by the results presented in his paper, the model chosen was inappropriate. Further comment is in order so that readers are not misled as to the generality of the conclusions presented in El-Khatib's paper.

The main concern is the indicated influence of mobility ratio on the model results, in which the communicating and noncommunicating cases were compared. These results are unrealistic. In reality, the critical influence is the positioning of the high-permeability layers. In a waterflood, if the high-permeability layers are at the top, the influence of gravity, if allowed to operate through vertical communication, will tend to mitigate the poor performance otherwise observed in the fully stratified case. Likewise, in the case where the high permeabilities are at the bottom, vertical communication will exacerbate the problem.[1] The spatial relationship of the permeabilities will control the relative performance of the communicating and noncommunicating cases, independent of the mobility ratio. In particular, the model's insensitivity to the existence of vertical communication, for the unit-mobility-ratio case, has to be an artifact of that model.

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(SPE 14490)

SPEJ

Author's Reply to Discussion of the Effect of Crossflow on Waterflooding of Stratified Reservoirs

N. El-Khatib, King Saud U.

Because of their belief that for stratified reservoirs "the spatial relationship of the permeabilities will control the relative performance of the communicating and noncommunicating cases, independent of the mobility ratio," Collins and Wang dispute the results of the model in the paper and describe the model as inappropriate. Their belief, however, is incorrect; the effect of mobility ratio is significant in all cases of waterflooding. Because of the different mobilities of the displacing and displaced fluids and the different velocities in the various layers of a stratified system, the pressure distribution in the different layers of a noncommunicating system will not be the same. If vertical communication between the layers is allowed, crossflow will take place from high-pressure zones to low-pressure zones so that the pressure at a given distance will be the same for all layers. This is the crossflow caused by viscous forces, and its influence is as described in the paper. It acts in such a way that it enhances the oil recovery for favorable mobility ratio and retards the recovery for unfavorable mobility ratio, unlike a noncommunicating system.

Gravitational and capillary forces also will affect crossflow between layers of communicating systems. Gravitational effect tends to increase the water saturation in the bottom layers while capillarity tends to reduce this effect to some degree by keeping water at locations of high potential. If the stratified system is such that the permeability increases continuously with depth, the additional effect of gravity will be to decrease the oil recovery; the gravitational effect will increase the oil recovery for systems in which permeability decreases continuously with depth. This change in oil recovery as a result of gravitational effect is relative to a communicating system with only viscous forces considered; it is not relative to a noncommunicating system, as Collins and Wang believe. Because the mobility ratio has a significant effect on both noncommunicating systems and communicating systems that ignore gravity, the net performance of communicating systems will be influenced by the mobility ratio. It must also be noted that the effect of gravity is at its maximum if the permeability is continuously increasing or decreasing with depth. For systems in which permeability increases and decreases alternately, the net effect of gravity on the performance tends to be minimized. The recovery increase for decreasing permeability with depth tends to compensate the recovery decrease for increasing permeability with depth. Because one expects the permeability distribution in the different layers to be random, neglecting the gravitational effect in the model can be justified.

Even for the permeability arrangements in which gravitational effect is at a maximum level, the gravitational effect can also be ignored if the ratio of gravitational to viscous forces is small. This ratio can be represented by the dimensionless group $\Delta\rho g\bar{k}^2 h/\mu_w VL (k_{max} - k_{min})$. Only a combination of high gravitational/viscous ratio and systematic permeability ordering will affect the predictions of the model seriously. Even then the performance still will be dependent on the mobility ratio.

(SPE 14692)

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Further Discussion of the Effect of Crossflow on Waterflooding of Stratified Reservoirs

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Addressing the "Author's Reply to Discussion of the Effect of Crossflow on Waterflooding of Stratified Reservoirs" (*Soc. Pet. Eng. J.*, Aug. 1985, Pages 614–15), we submit the following further discussion.

At issue is the relative importance of gravitational and viscous forces in explaining the behavior of the communicating and noncommunicating cases in waterflooding a stratified reservoir. Our position is that, in the communicating case, gravitational forces would dominate, with good or bad effects depending on the spatial permeability variation.[1] El-Khatib's mathematical model cannot address this issue because it does not account for gravitational forces. In the absence of gravity, the dominance of the viscous forces produced what we considered to be unrealistic results from the model. Because El-Khatib rejects our belief, and little can be gained from a review of the literature, the reader may find it difficult to form an opinion about the subject. Craig[2] discusses most of the pertinent literature. From Craig's references, one can find support for both sides of the argument on the relative importance of mobility ratio and gravity in waterflooding stratified reservoirs. Perhaps the most interesting is Hutchinson's[3] conclusion, which is based on preliminary results: "Thus we can calculate reservoir performance on the basis of noncommunicating sands when the mobility ratio is one or greater (unfavorable)."

Therefore, we submit results obtained with a numerical simulator, the Intercomp Beta II™, to support our position. Communicating and noncommunicating cases were compared with the use of the permeability configuration in El-Khatib's paper, the highest at the top and decreasing toward the bottom. (This permeability arrangement is characteristic of coarsening-upward sand sequences—such as the Lower Sparky formation of Alberta and Saskatchewan.[4]) We used real rock and fluid properties and capillary pressure data in the models. The effects of the latter, however, were minor. The mobility ratio was varied by adjustment of the oil viscosity.

The results are shown in Fig. D-1. Contrary to El-Khatib's results, these show that, for the given permeability configuration, the communicating case performs better, independent of mobility ratio. This does not mean that mobility ratio does not play a role but rather that it is minor relative to gravity in the two communication comparisons. Absolute comparisons with the El-Khatib paper are not intended because we used a different system.

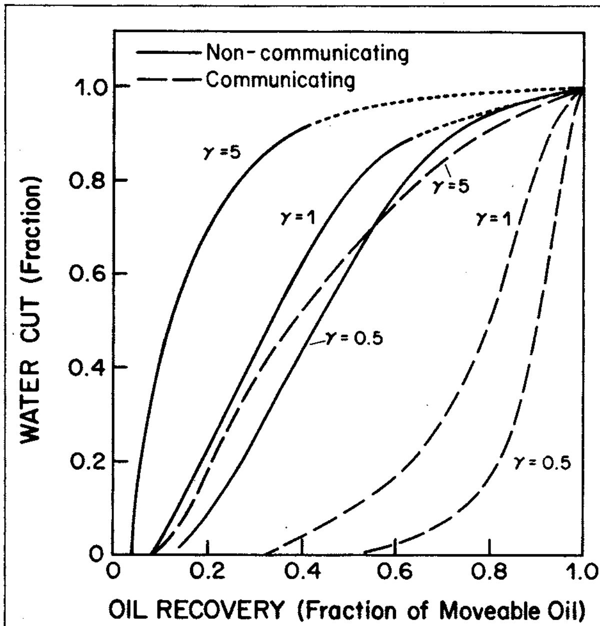

Fig. D-1—Comparison of communicating and noncommunicating cases for three mobility ratios (γ) with the SSI Beta II simulator. Grid dimensions are 10 × 1 × 12.

Numerical simulators are based on mathematical models that may be invalid. We hope that this discussion will promote additional efforts to validate numerical simulators with laboratory data and vice versa with respect to stratified waterfloods at unfavorable mobility ratios.

References

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Author's Reply to Further Discussion of the Effect of Crossflow on Waterflooding of Stratified Reservoirs

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The flow equations describing two-phase immiscible flow in porous media demonstrate clearly that the performance of a stratified system is influenced by both viscous and gravitational forces. In their first discussion (*Soc. Pet. Eng. J.*, Aug. 1985, Page 614), Collins and Wang claimed that "the spatial relationship of the permeabilities will control the relative performance of the communicating and noncommunicating cases, independent of the mobility ratio." In their "Further Discussion," they claimed that the mobility ratio is "minor relative to gravity in the two-case communication comparison." To support their claim, they presented results obtained with a numerical simulator for a case when permeability continuously decreased with depth and observed that the communicating case performs better than the noncommunicating case, independent of mobility ratio. My responses follow.

To include both viscous and gravitational forces in a mathematical model and still get an analytical solution to the problem is not possible. This is the case in other mathematical models[1-4] where the gravitational forces are ignored.

For noncommunicating systems,[1,2] the relative location of the different layers is immaterial because crossflow between layers is not allowed.

For communicating systems, the performance relative to noncommunicating systems depends on both viscous and gravitational forces. The effect of mobility ratio alone, as predicted by the model in my paper, is in agreement with other numerical[4] and experimental[5] results.

The influence of gravity in all cases of permeability distribution is to increase the water saturation in the bottom layers and to decrease it in the top layer. The effect of this gravitational segregation on reservoir performance will depend on the permeability/depth configuration. For cases where permeability increases with depth, which causes the water front to advance faster in the bottom layers, the increase in water saturation at the bottom as a result of gravity will worsen the performance of the system. On the other hand, if the permeability decreases with depth, the effect of gravity is to sharpen the displacement front and thus to improve the performance. The concern now is the effect of gravity on the performance when the permeability alternately increases and decreases with depth, as is the case in actual reservoirs. Because the effect of gravity on the increasing and decreasing permeability distributions is in opposite directions, we logically assume that some permeability distributions exist for which the effect of gravity would be zero. It is also reasonable to assume that there are other random permeability distributions for which the effect of gravity is negligible. In any case, the effect of gravity for random permeability distribution obviously will not be as severe as for the cases of continuously increasing or decreasing permeabilities. This reasoning justifies neglecting gravity in the different mathematical models.

Collins and Wang gave the false impression that the permeability configuration of their example is the same as that in my paper. The example in my paper is taken from Warren and Cosgrove,[4] where the permeability has a log-normal distribution. In all analytical models for stratified systems, the permeability is arranged in a decreasing order as a convenient computational procedure only.

Therefore, the results presented by Collins and Wang clearly do not justify their claim of a minor role of mobility ratio relative to gravitational forces in the two-case communication comparisons. To prove that, they should show the same for randomly distributed permeability stratifications as in actual reservoirs. Some actual field data are reported in the literature.[6,7]

Fig. R-1 summarizes the effect of both gravitational and viscous forces on the performance of communicating systems relative to that of noncommunicating systems for different cases of mobility ratios and permeability configurations. The figure shows that gravity and mobility ratio effects may work in the same direction or in opposite directions. When gravity works in the opposite direction from the mobility ratio, the net performance of the system will depend on the relative effects of the two forces. Only for severe conditions will gravity effect override the mobility ratio effect and reverse the performance trend predicted by the mathematical model. It is not true that for all permeability configurations with permeabil-

Gravity \ Mobility ratio			$\gamma < 1$ +	$\gamma = 1$ 0	$\gamma > 1$ −
Permeability Configuration	Decreasing	+	+	+	?
	Increasing	−	?	−	−
	Random	m	+	m	−

\+ Communicating system performs better

\- Communicating system performs worse

m Minor effect

Fig. R-1–Effect of gravity and mobility ratio on the performance of communicating relative to noncommunicating reservoirs.

ity decreasing with depth, the performance of the communicating system would be better than that of the noncommunicating for unfavorable mobility ratios ($\gamma > 1$). If the permeability contrast between the maximum and minimum is decreased, the performance of the system will be changed.

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